A Field-Particle Correlation Analysis of a Continuum Vlasov-Maxwell Perpendicular Collisionless Shock

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Collaborators

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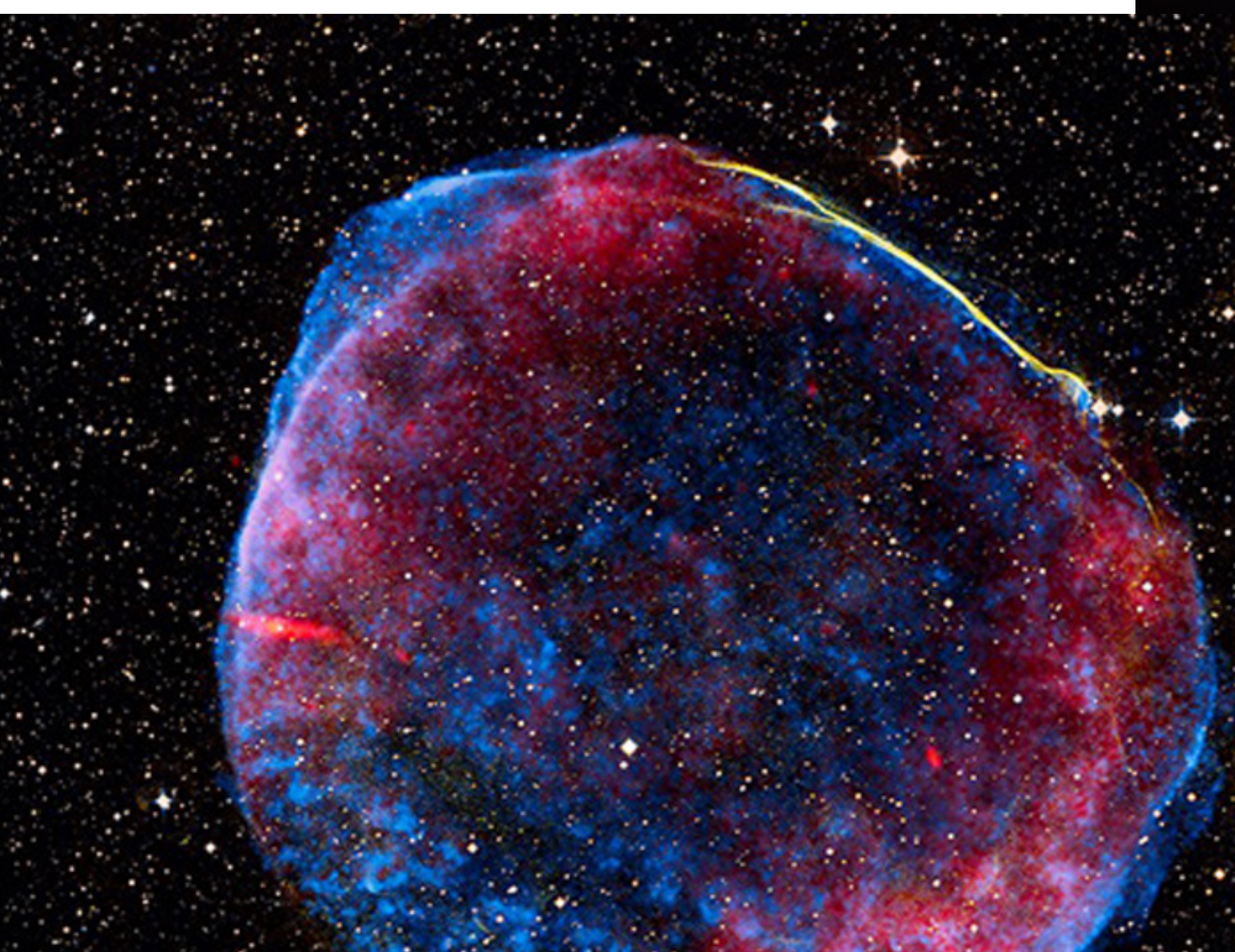
Outline

- Introduce shocks

- Introduce Gkeyll and the field-particle correlation

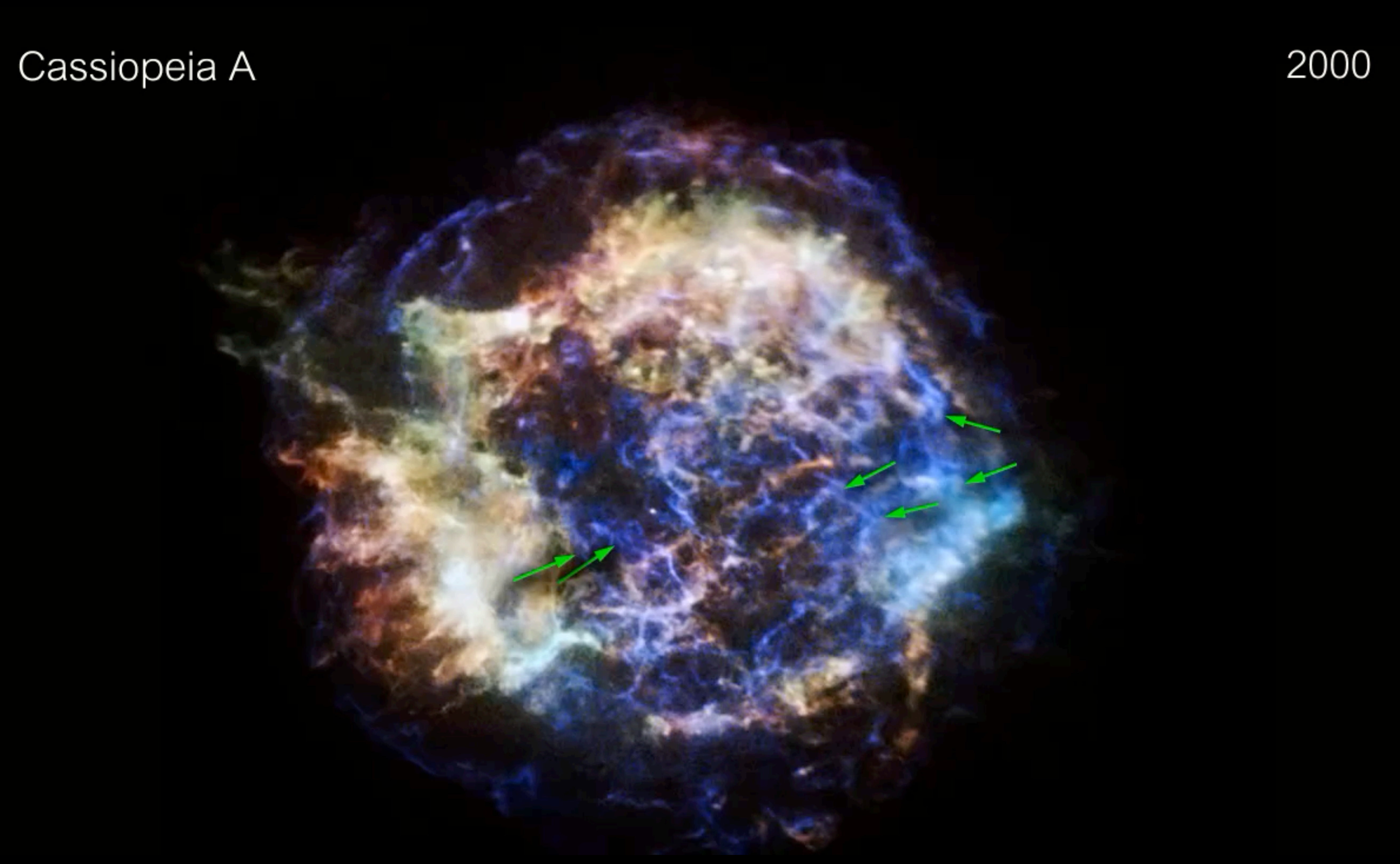
- Present shock results, including ion and electron energization in phase space



What is a shockwave?

$v = 0, c$

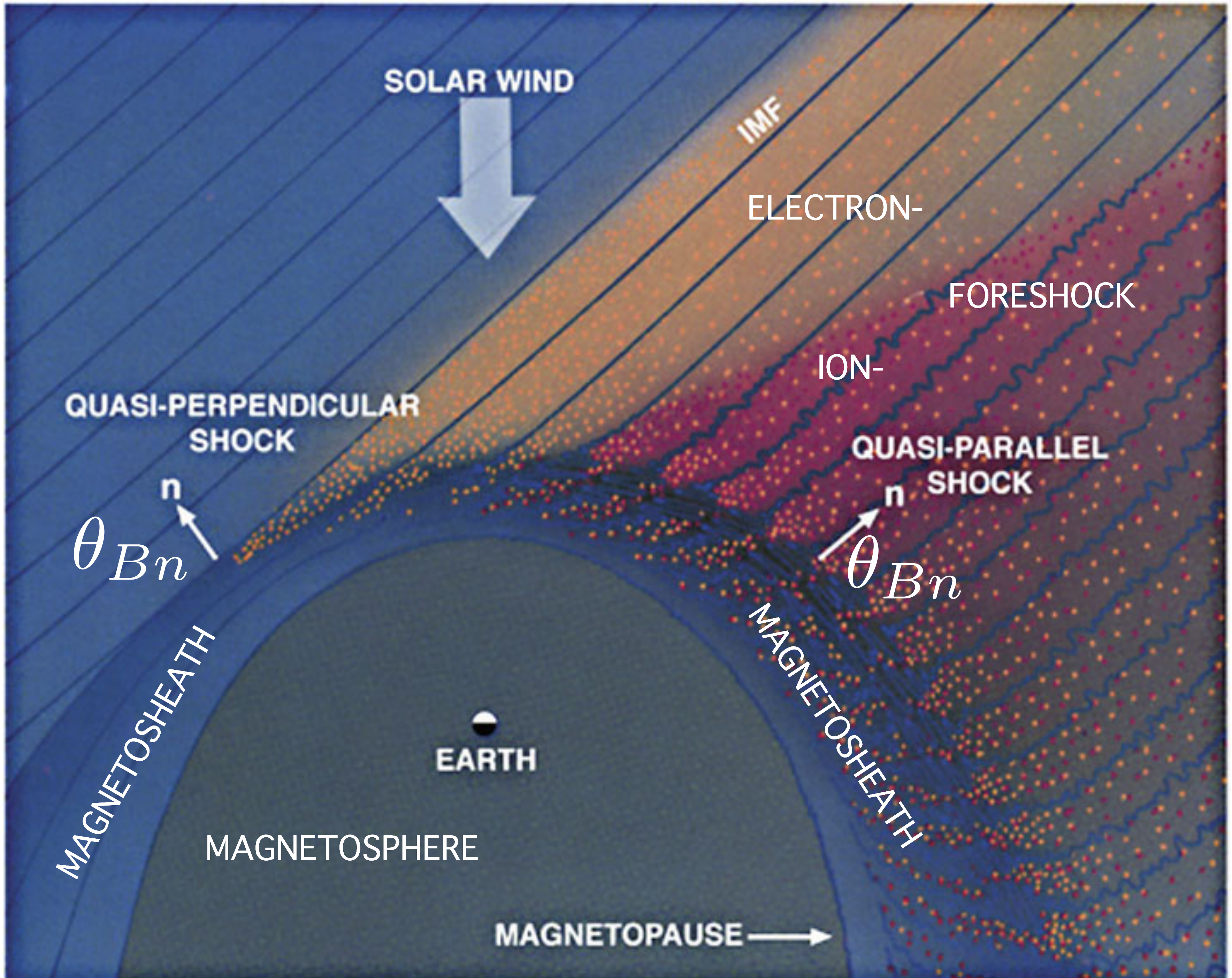
Supernova Timelapse (X-ray data from Chandra)



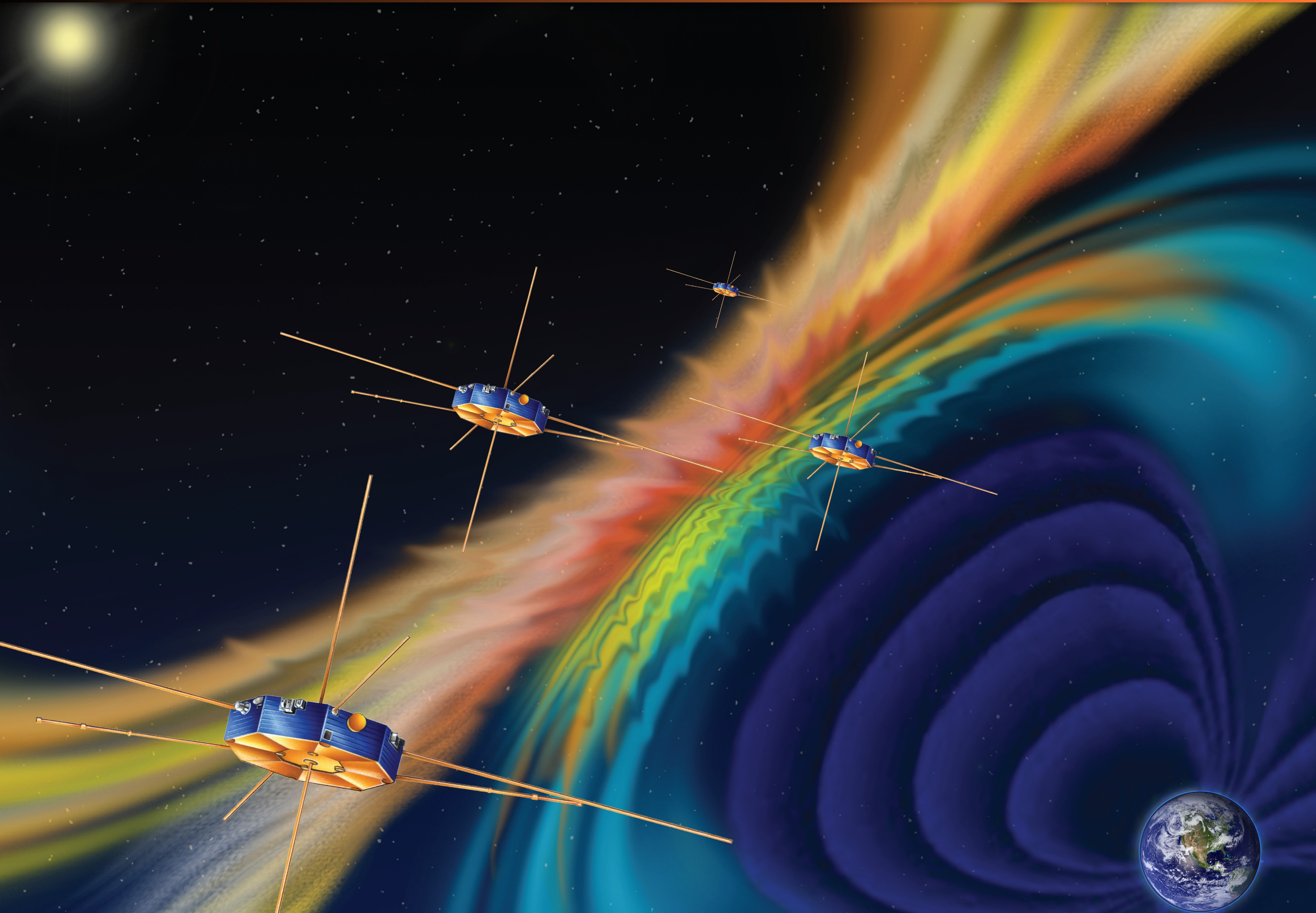
Parameterizing a Shock

In space and astrophysical plasmas, shocks are typically nonlinearly steepened fast magnetosonic-whistler waves

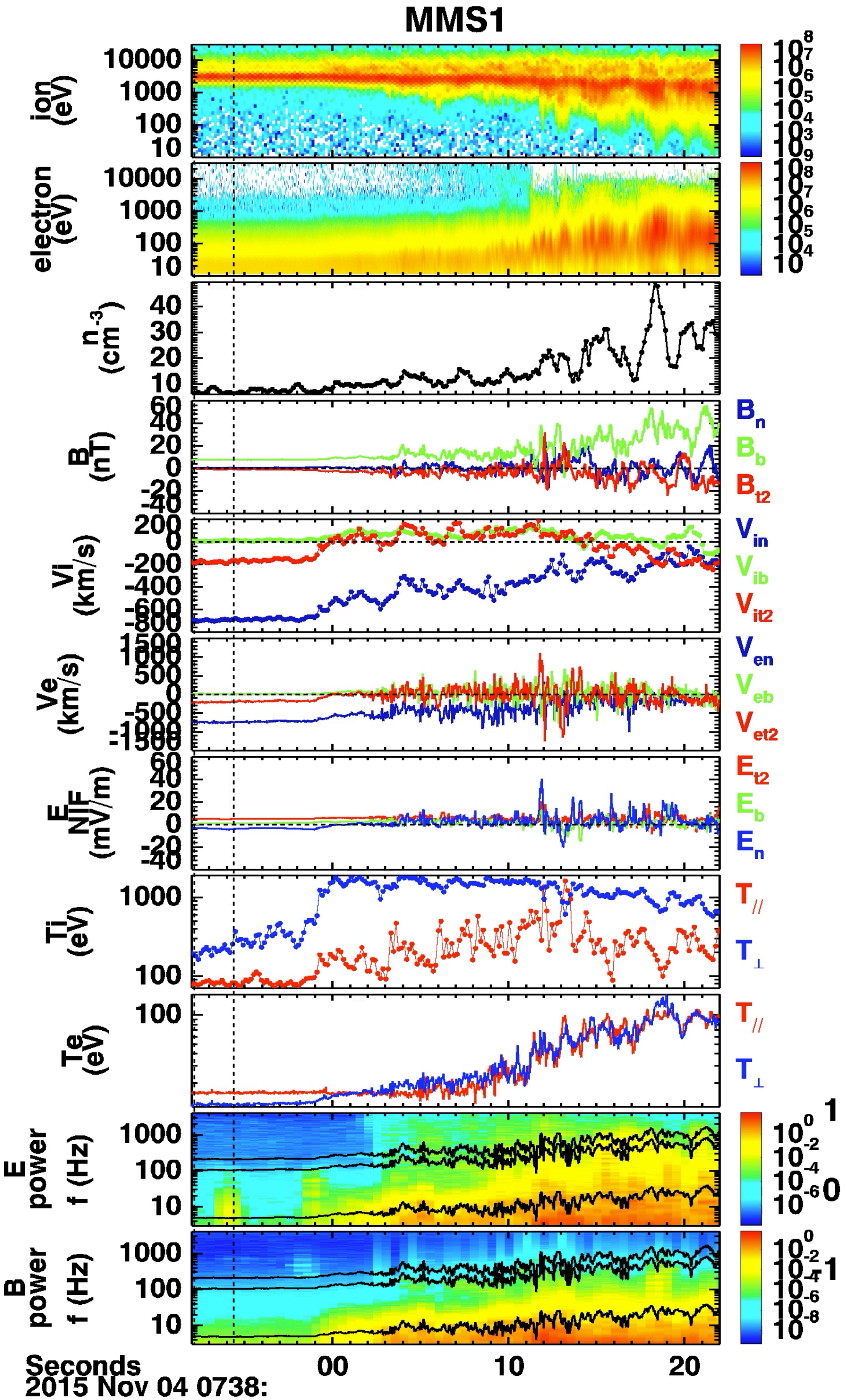
Most important parameters for a shock: $\beta, M_s = U_{shock}/v_{ms}, \theta_{Bn}$

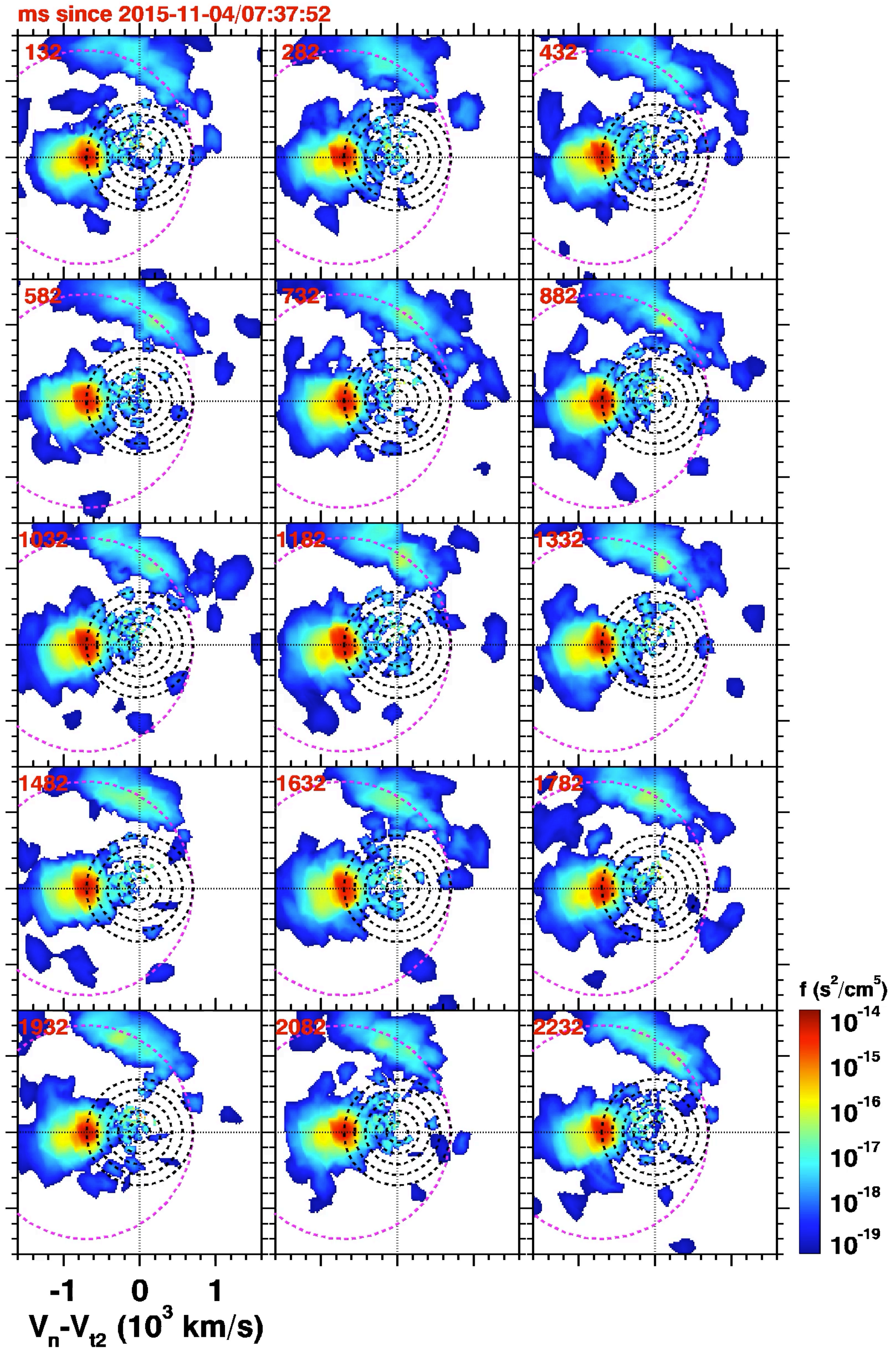


Magnetospheric Multiscale (MMS) Mission



In Situ MMS Data for Perpendicular (89.6º) Shock





Gkeyll Simulation Framework

The Gkeyll (and Hyde) framework*

"It is one thing to mortify curiosity, another to conquer it."

- The Gkeyll framework is flexible suite of solvers for plasma physics being developed at the Princeton Plasma Physics Lab, UMD, Virginia Tech, and MIT
- Solvers include a finite volume method for equations written in conservative form and a discontinuous Galerkin finite element method for systems of equations which can be written in terms of a Poisson bracket
- Multiple Vlasov-Maxwell publications already:
  - P. Cagas, A. Hakim, J. Juno, B. Srinivasan, Continuum kinetic and multi-fluid simulation of a classical sheath, *Phys. Plasmas* (2017).
  - P. Cagas, A. Hakim, B. Srinivasan, Nonlinear saturation of the Weibel instability, *Phys. Plasmas* (2017).
  - **J. Juno, A. H. Hakim, J. M. TenBarge, B. Dorland, E. L. Shi, Discontinuous Galerkin algorithms for fully kinetic plasmas. *JCP* (2018).**
  - I. Pusztai, J. M. TenBarge, A. N. Csapo, J. Juno, A. Hakim, L. Yi, T. Fülöp, Low Mach-number collisionless electrostatic shocks and associated ion acceleration, PPCF (2018)
  - V. Skoutnev, A. Hakim, J. Juno, J. M. TenBarge, Temperature Dependent Saturation of Weibel Type Instabilities in Counter-Streaming Plasmas, ApJL (2019)
  - A. Sundström, J. Juno, J. M. TenBarge, and I. Pusztai. Effect of a weak ion collisionality on the dynamics of kinetic electrostatic shocks, JPP (2019)
  - I. Pusztai, J.. Juno, A. Brandenburg, J. M. TenBarge, A. Hakim, M. Francisquez, and T. Fülöp. Dynamo in weakly collisional non-magnetized plasmas impeded by Landau damping of magnetic fields, PRL (2020)
  - **A. Hakim and J. Juno, Alias-free, matrix-free, and quadrature-free discontinuous Galerkin algorithms for (plasma) kinetic equations, Supercomputing (2020).**
  - **A. Hakim, M. Francisquez, J. Juno, G. W. Hammett, Conservative Discontinuous Galerkin Schemes for Nonlinear Fokker-Planck Collision Operators, JPP (2020)**

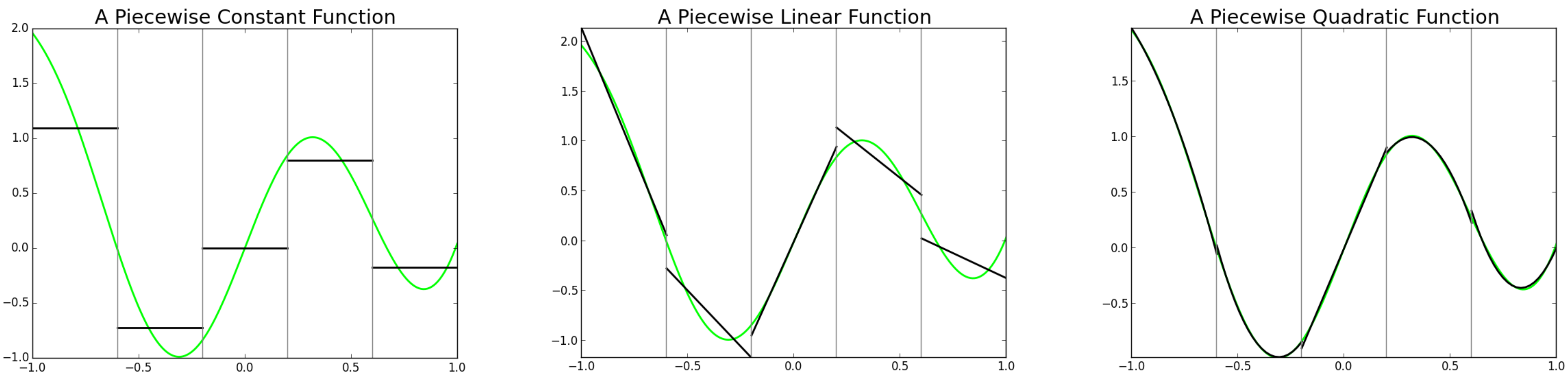
*https://github.com/ammarhakim/gkyl
https://gkyl.readthedocs.io/en/latest/

The discontinuous Galerkin finite element method

We choose to use the discontinuous Galerkin framework to discretize the full phase space of the Vlasov-Maxwell system because it combines aspects of

-Finite elements: high order accuracy and ability to handle complicated geometries

-Finite volume: locality of data and stability enforcing limiters



The discrete Vlasov equation

- What does the discontinuous Galerkin discretization of the Vlasov equation look like?

$$\frac{\partial f_s}{\partial t} + \nabla \cdot (\mathbf{v} f_s) + \nabla_v \cdot (\mathbf{F}_s f_s) = 0$$

  - Consider a phase space mesh $\mathcal{T}$ with cells $K_j \in \mathcal{T}, j = 1, \ldots, N$.
  - Then the problem formulation is, find $f_h \in \mathcal{V}_h^p$, such that for all $K_j \in \mathcal{T}$,

$$\int_{K_j} w \frac{\partial f_h}{\partial t}\, d\mathbf{z} + \oint_{\partial K_j} w^- \mathbf{n} \cdot \hat{\mathbf{F}}\, dS - \int_{K_j} \nabla_z w \cdot \alpha_h f_h\, d\mathbf{z} = 0$$

$$f_h(\mathbf{z}, t) = \sum_n^{N_p} F_n(t) w_n(\mathbf{z}) \qquad \mathcal{V}_h^p = \{v : v|_{K_j} \in \mathbf{P}^p, \forall K_j \in \mathcal{T}\},$$

<u>Conserves</u>
Number density
Energy
L2 norm of the distribution decays monotonically

<u>Cost Mitigation</u>
Lua-JIT & C++
Computer algebra pre-generated kernels
MPI + MPI-3 shared memory
Reduced and orthonormal basis sets

Field-Particle Correlations

Field-particle correlations defined [Howes, Klein, & Li (2017)]

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_x f + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f = 0$$

$$f = f_0 + \delta f$$

Separation useful in some cases but not necessary

Multiply by $mv^2/2$ and integrate to obtain the energy equation

$$\frac{\partial W}{\partial t} = \int \int d\mathbf{x} d\mathbf{v} \frac{1}{2} m v^2 \left[-\mathbf{v} \cdot \nabla_x f - \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f \right]$$

$$\frac{\partial W}{\partial t} = \int \int d\mathbf{x} d\mathbf{v} \frac{1}{2} m v^2 \frac{\partial f}{\partial t}$$

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$$\frac{\partial W}{\partial t} = \int \int d\mathbf{x} d\mathbf{v} \frac{1}{2} m v^2 \frac{\partial f}{\partial t}$$

$$\frac{\partial W}{\partial t} = -\frac{q}{2} \int \int d\mathbf{x} d\mathbf{v} v^2 \mathbf{E} \cdot \nabla_v f = q \int \int d\mathbf{x} d\mathbf{v} \mathbf{E} \cdot \mathbf{v} f = \int d\mathbf{x} \mathbf{J} \cdot \mathbf{E}$$

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The field-particle correlation: $C(\mathbf{v}, t, \tau) = C_\tau(-q\frac{v^2}{2}\nabla_v f, \mathbf{E})$

In discrete form $C(\mathbf{v}, t_i, \tau) = -\frac{1}{N} \sum_{j=i}^{i+N} q \frac{v^2}{2} \mathbf{E}_j \cdot \nabla_v f_j$

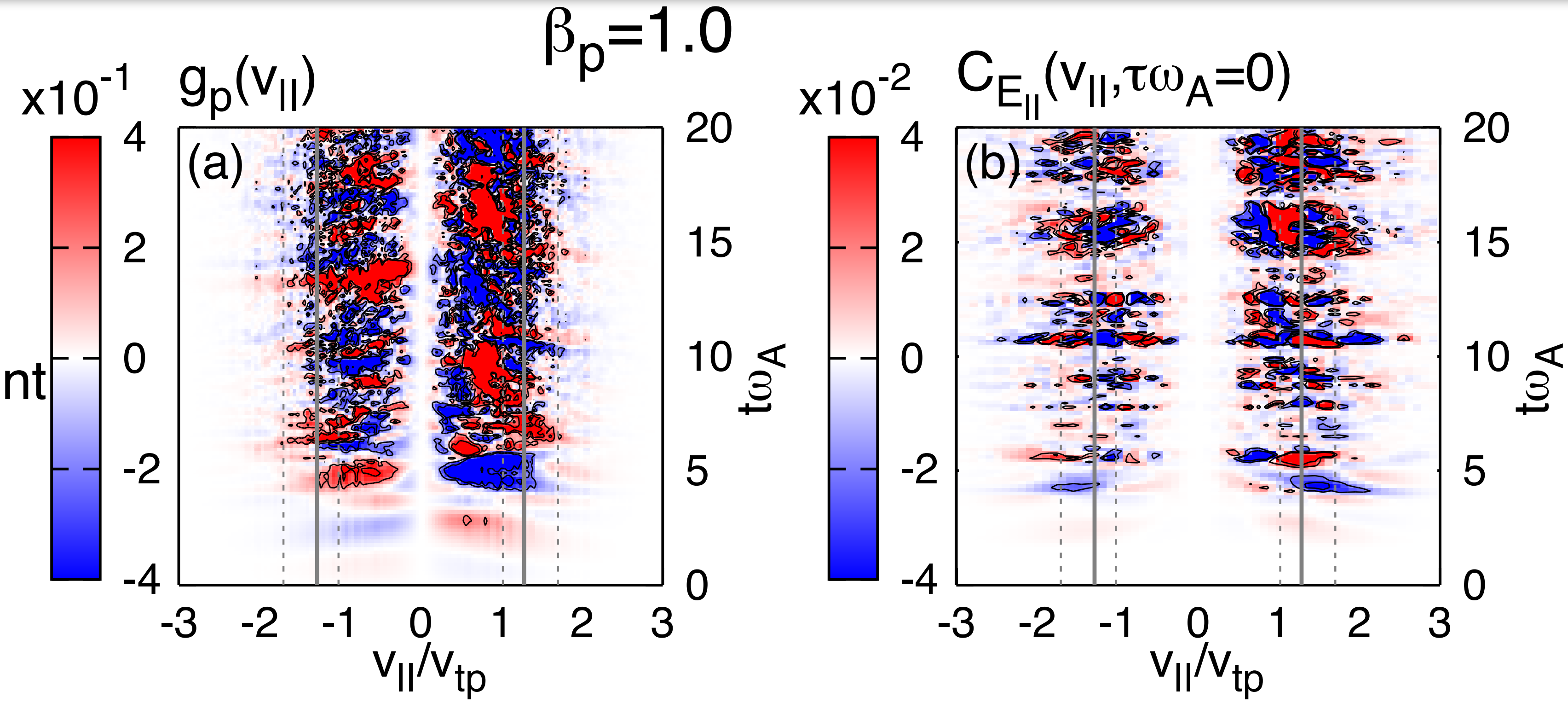
$t_j \equiv t(j\Delta t)$ $\qquad \tau = N\Delta t,$

Note that f or δf can be used

Note that the time average is an optional procedure that is used to remove oscillating, non-secular, energy exchange, which is very useful in turbulent systems.

Field-particle correlations in turbulence [Klein et al JPP (2017)]

Distribution function has no significant coherent structure



FPC with τ=0 displays sloshing of energy to and from the particles

The single point field particle correlation of a 3x2v gyrokinetic turbulence simulation. From Klein et al JPP (2017)

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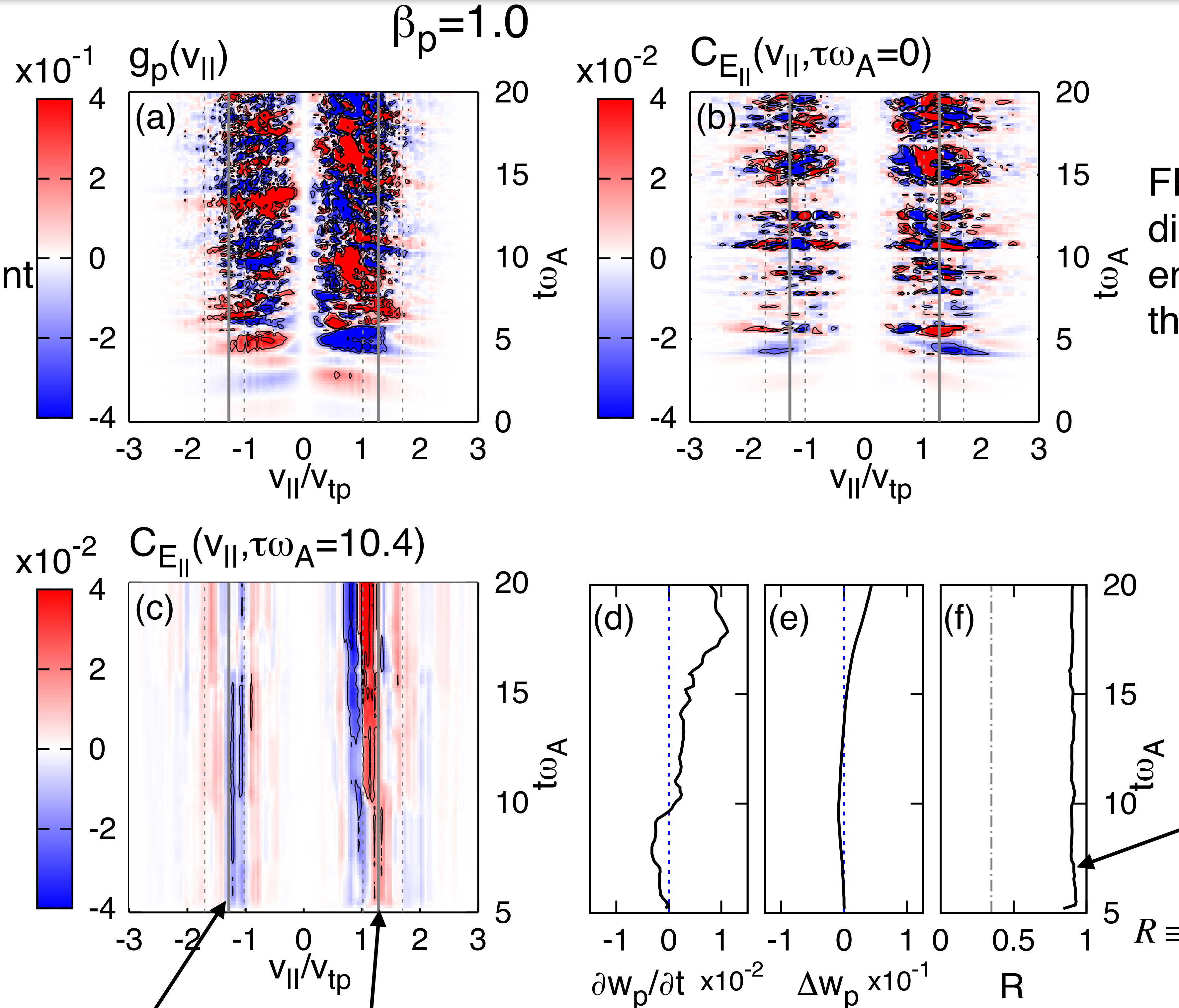


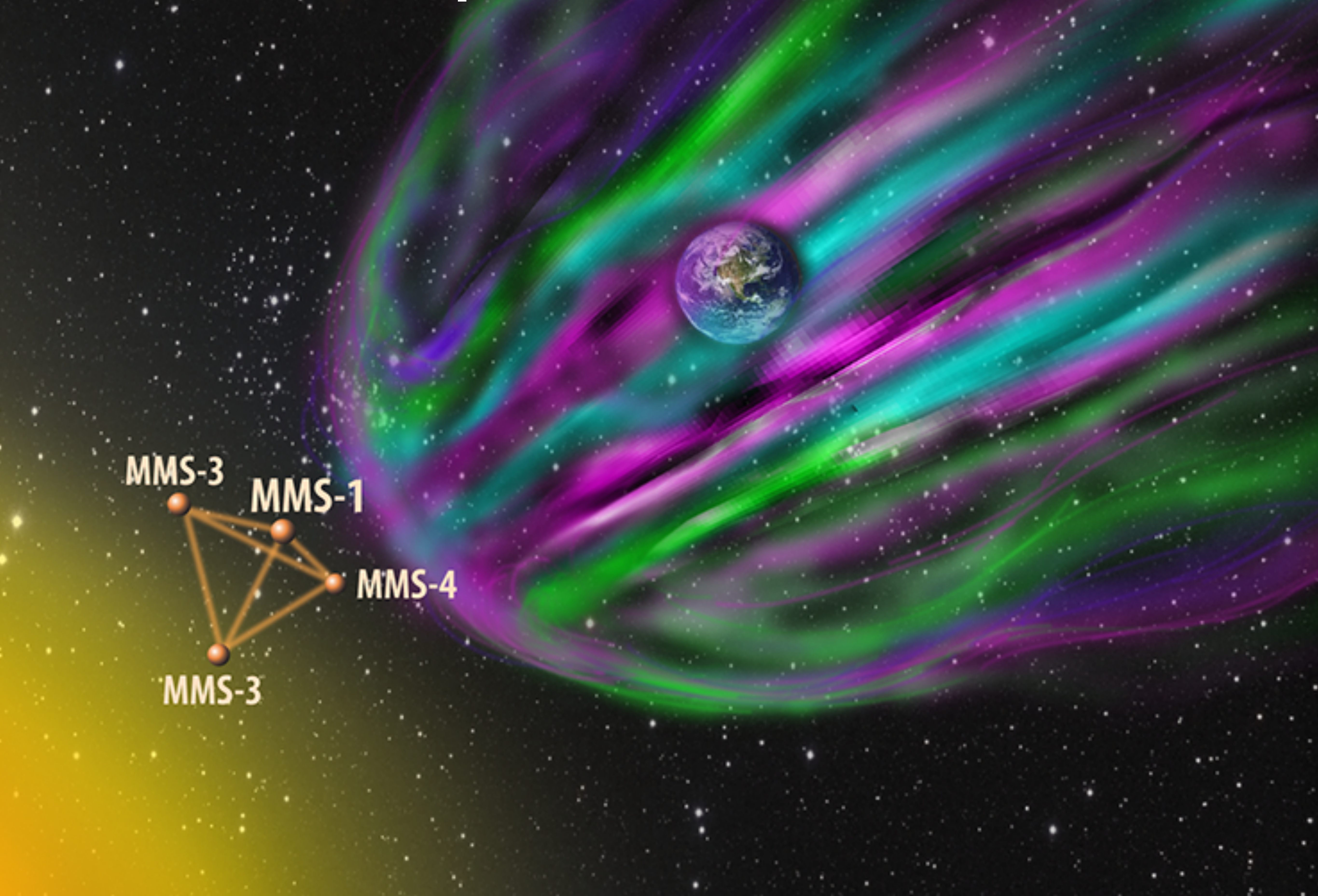
> 90% of the energy dissipation occurs in the resonant band

$$R \equiv \frac{\int_{v_1}^{v_2} \mathrm{d}v_\parallel |C_{E_\parallel}(v_\parallel)|}{\int_{-4v_{tp}}^{4v_{tp}} \mathrm{d}v_\parallel |C_{E_\parallel}(v_\parallel)|}$$

Finite time correlation recovers secular particle energization in the vicinity of the Landau resonant velocities

The single point field particle correlation of a 3x2v gyrokinetic turbulence simulation. From Klein et al JPP (2017)

Perpendicular shock



Weakly Collisional Perpendicular Shock Gkeyll Simulation Setup

Motivated by MMS observation provided by Chen and Wang

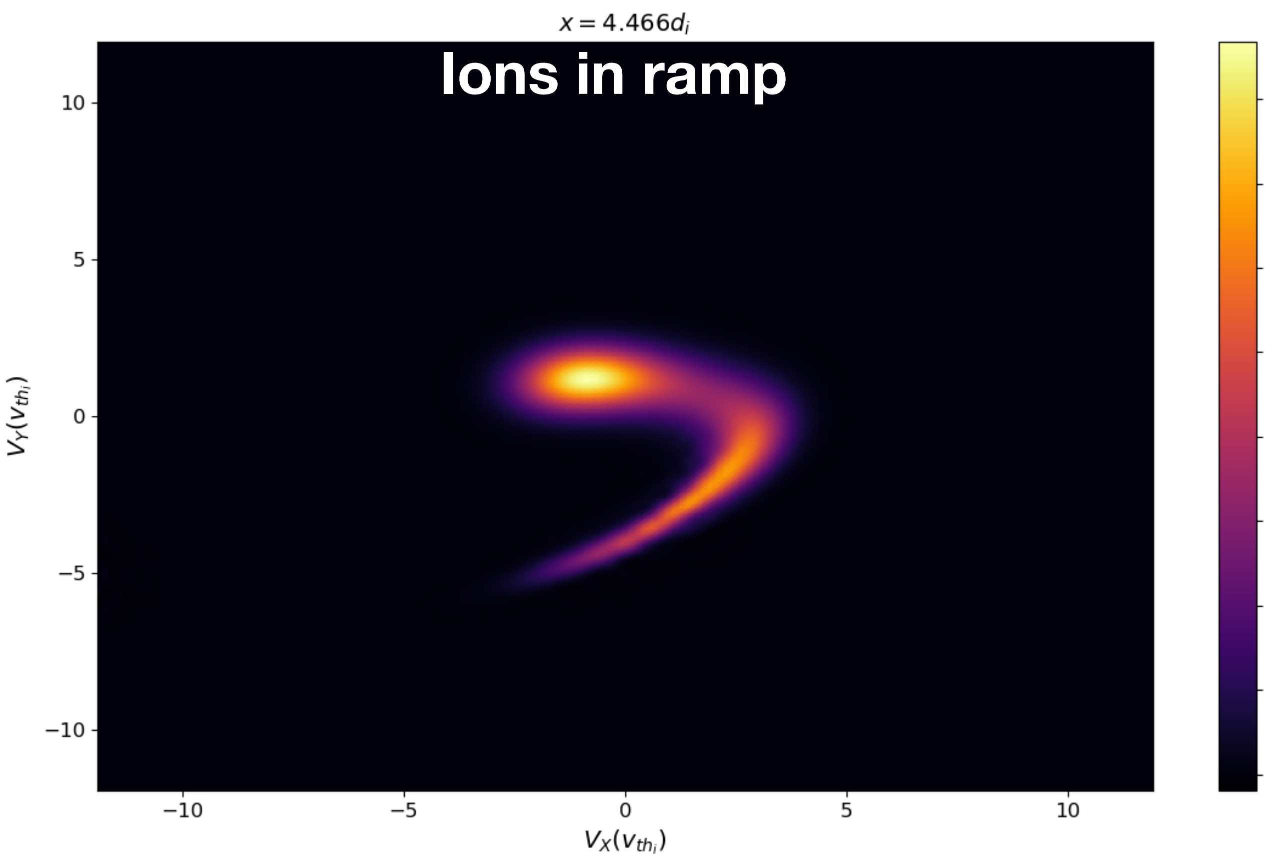
$$M_s \simeq 3, B = B_0\hat{\mathbf{z}}$$

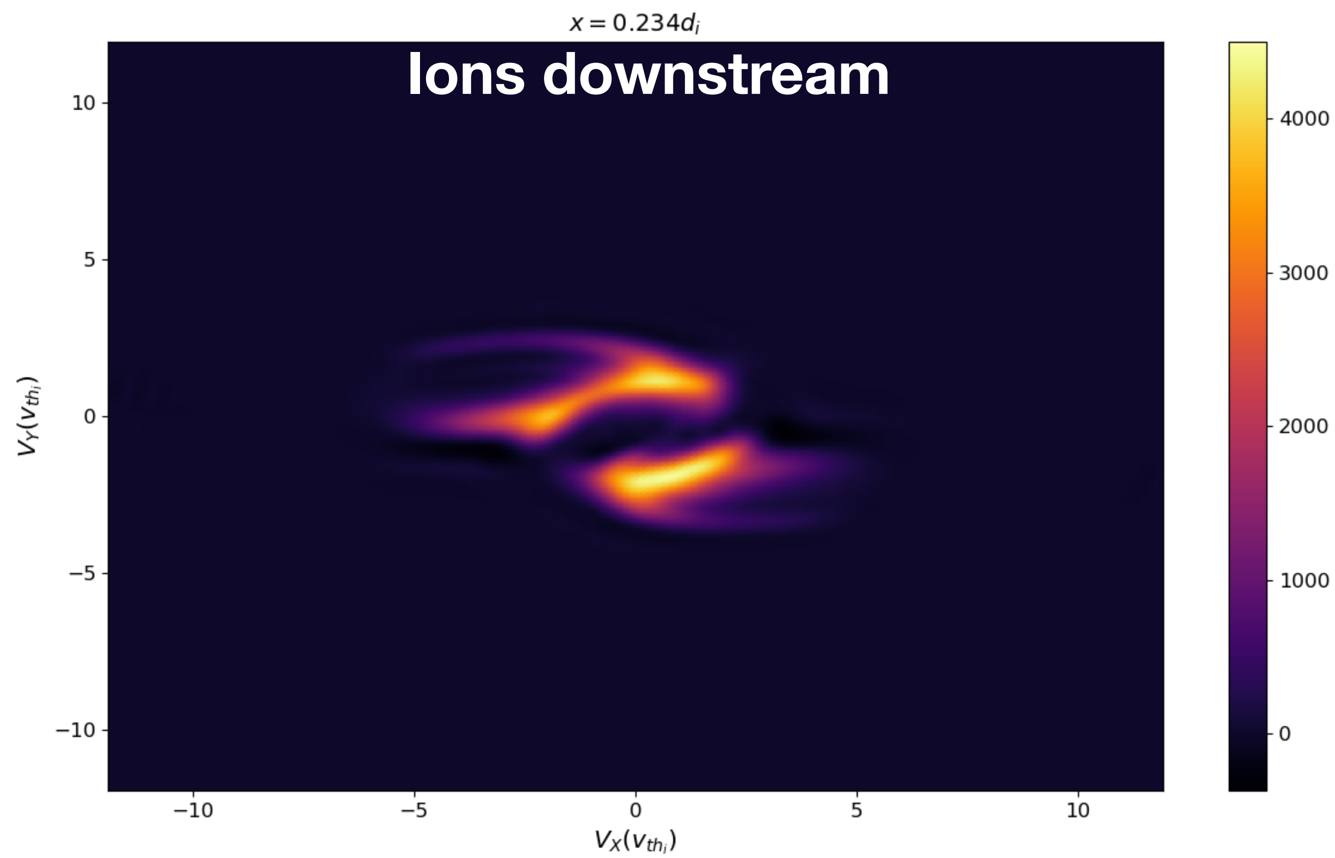
$$m_p/m_e = 100, \beta_p = 1.3, \beta_e = 0.7, v_{te}/c = 1/16$$

$$L_x = 25d_p, |v^p_{max}/v_{tp}| = 12, |v^e_{max}/v_{te}| = 6$$

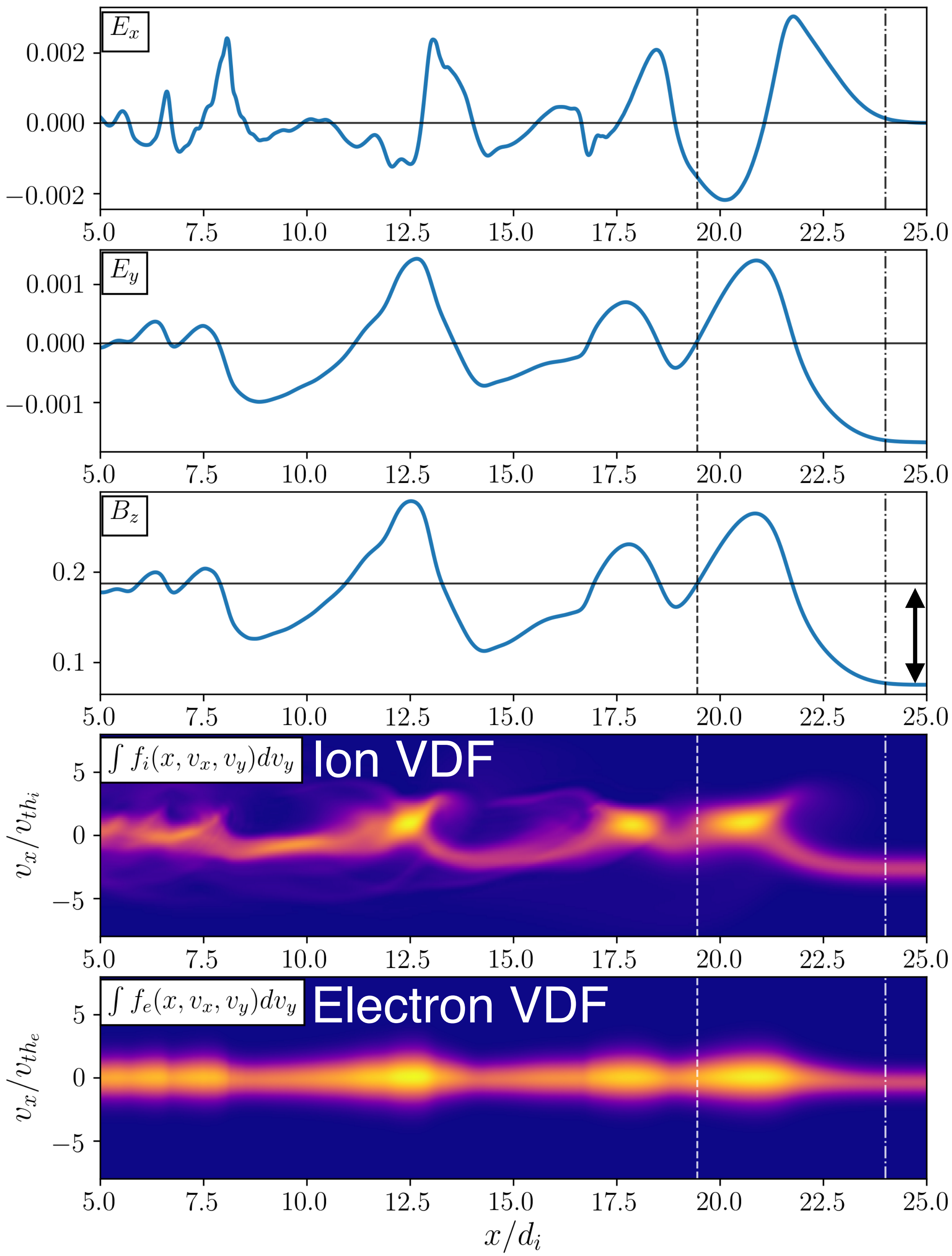
$$(n_x, n_v^2) = (1536, 48^2),\ p = 2$$

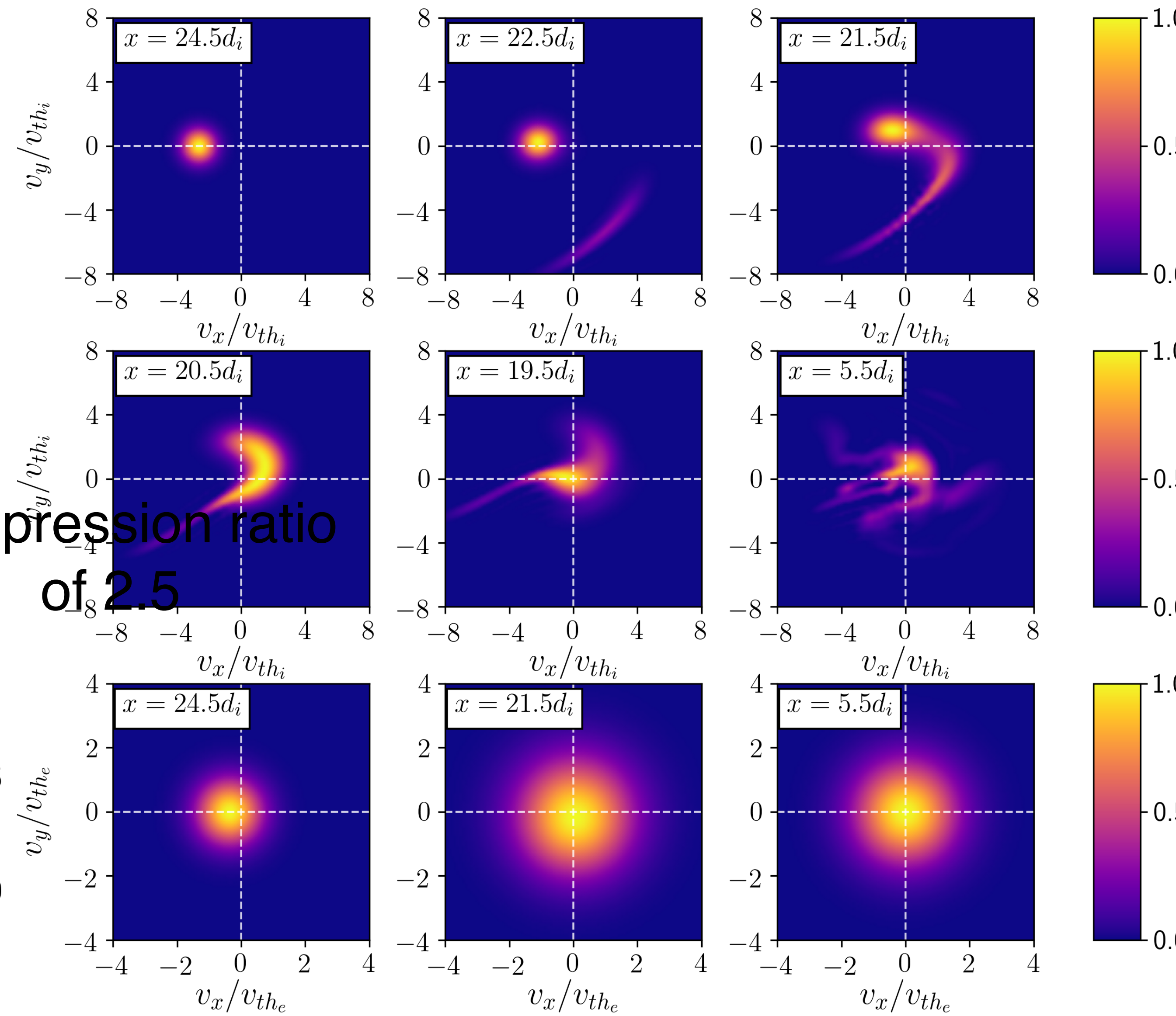
$$\nu_{ee} = 0.0001\Omega_{ce}$$





Weakly Collisional Perpendicular Shock Simulation Evolution

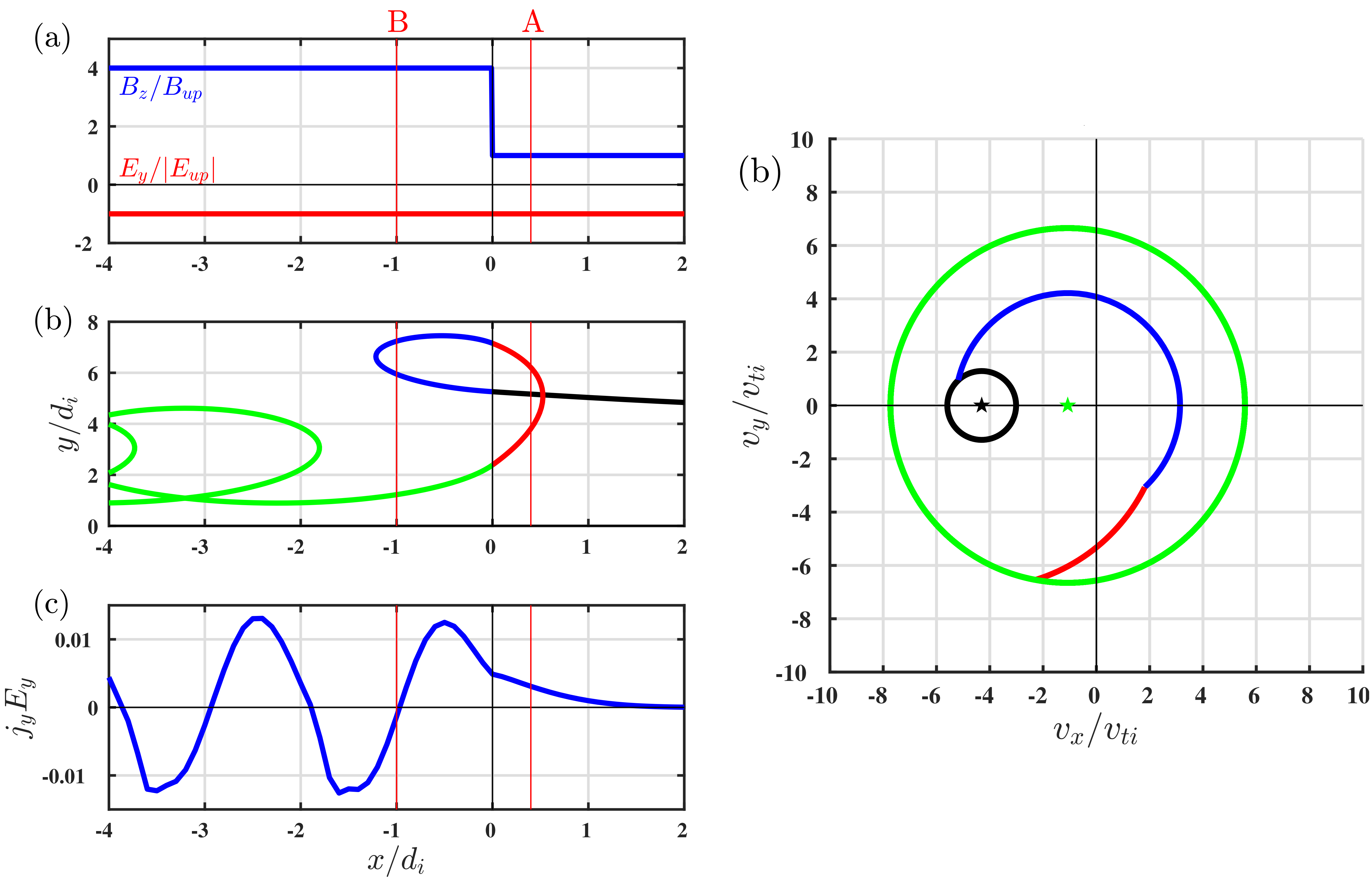




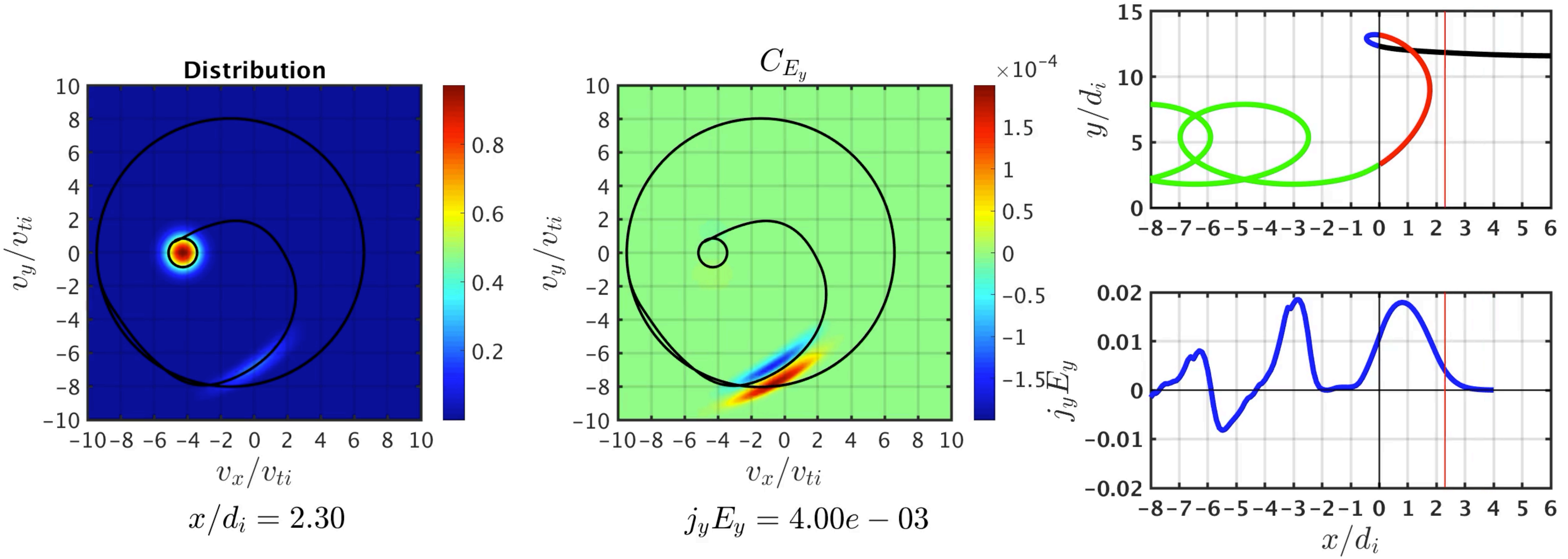
The ion (top two rows) and electron (bottom row) distribution functions plotted through the shock.

Ion energization

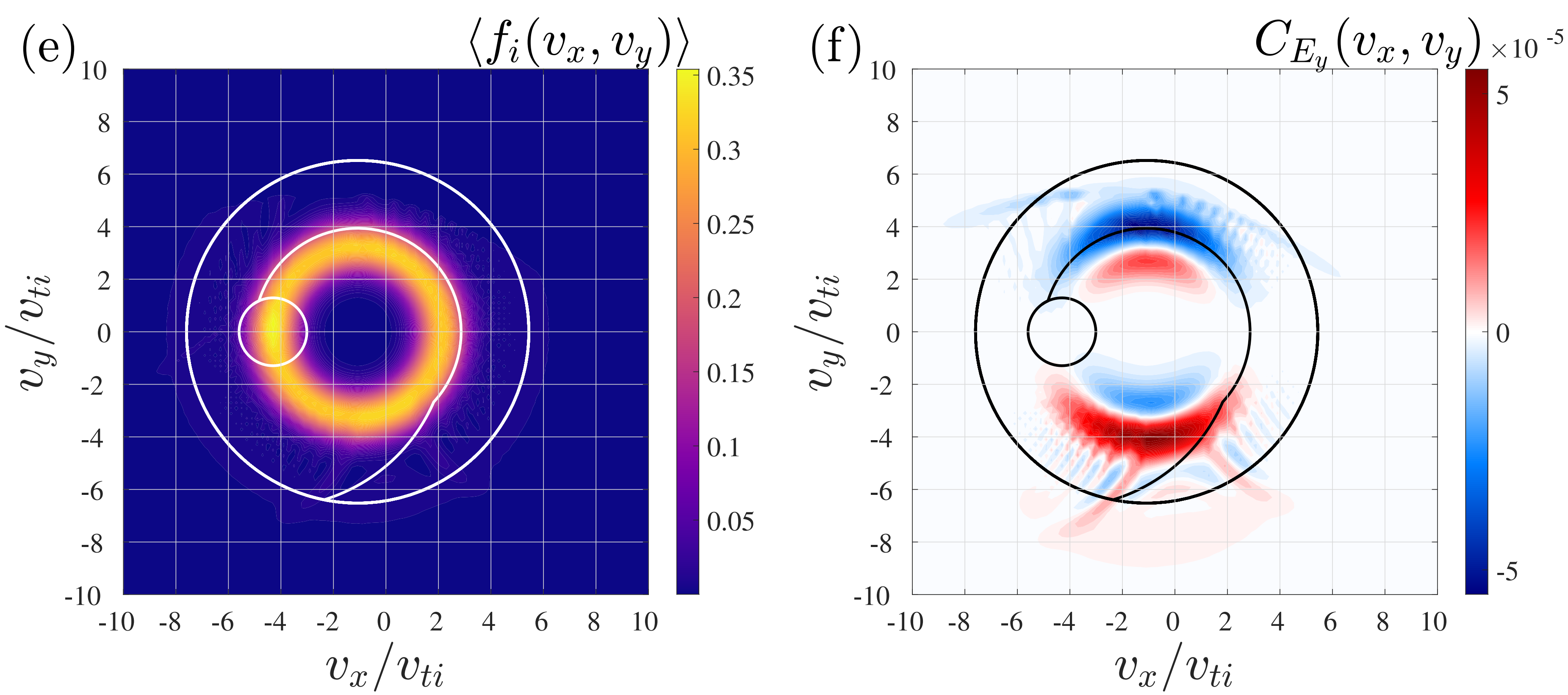
Simple Perpendicular Ion Shock Model, Single Particle Motion



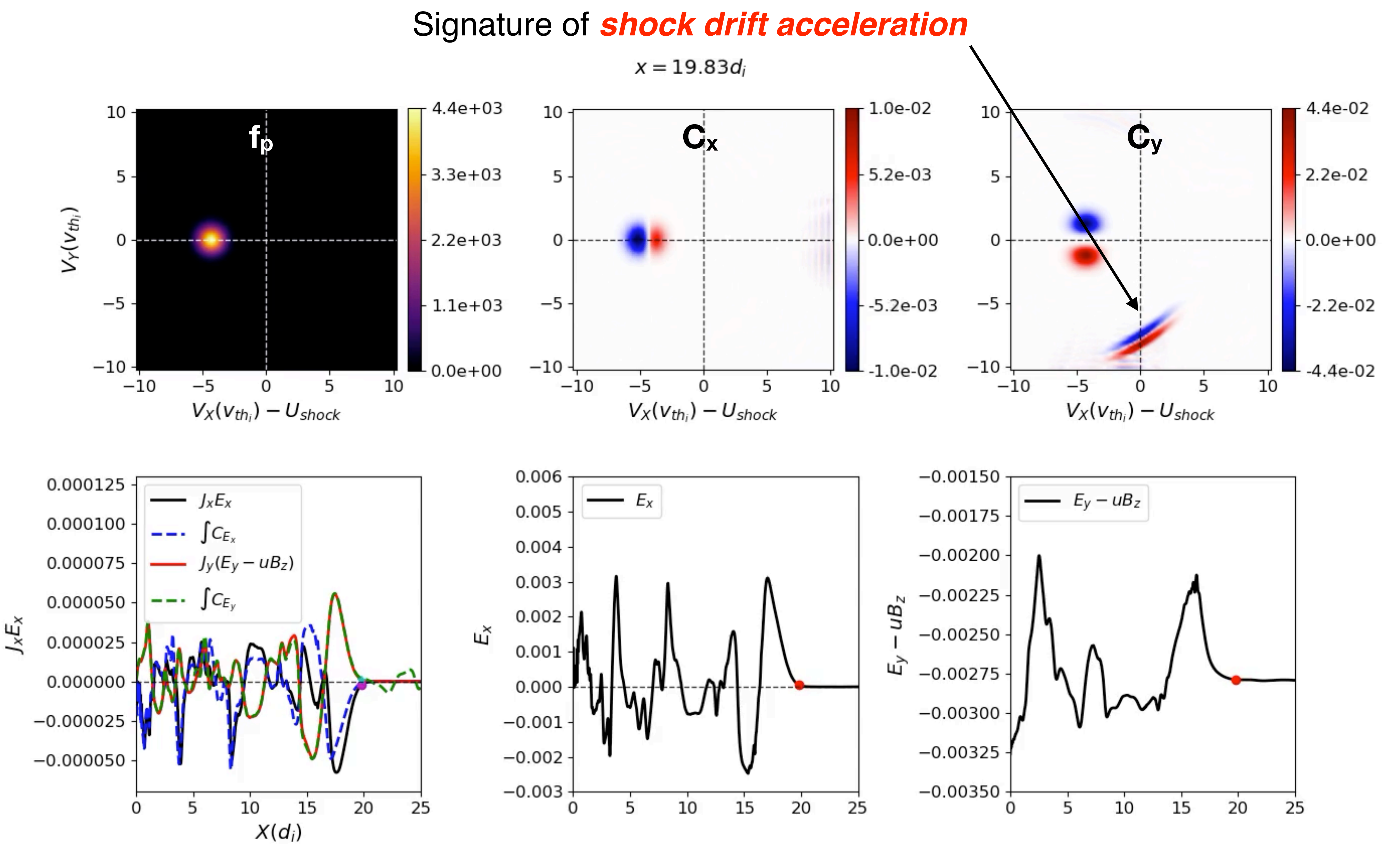
Perpendicular Ion Shock Model Field-Particle Correlation



Averaged over the downstream

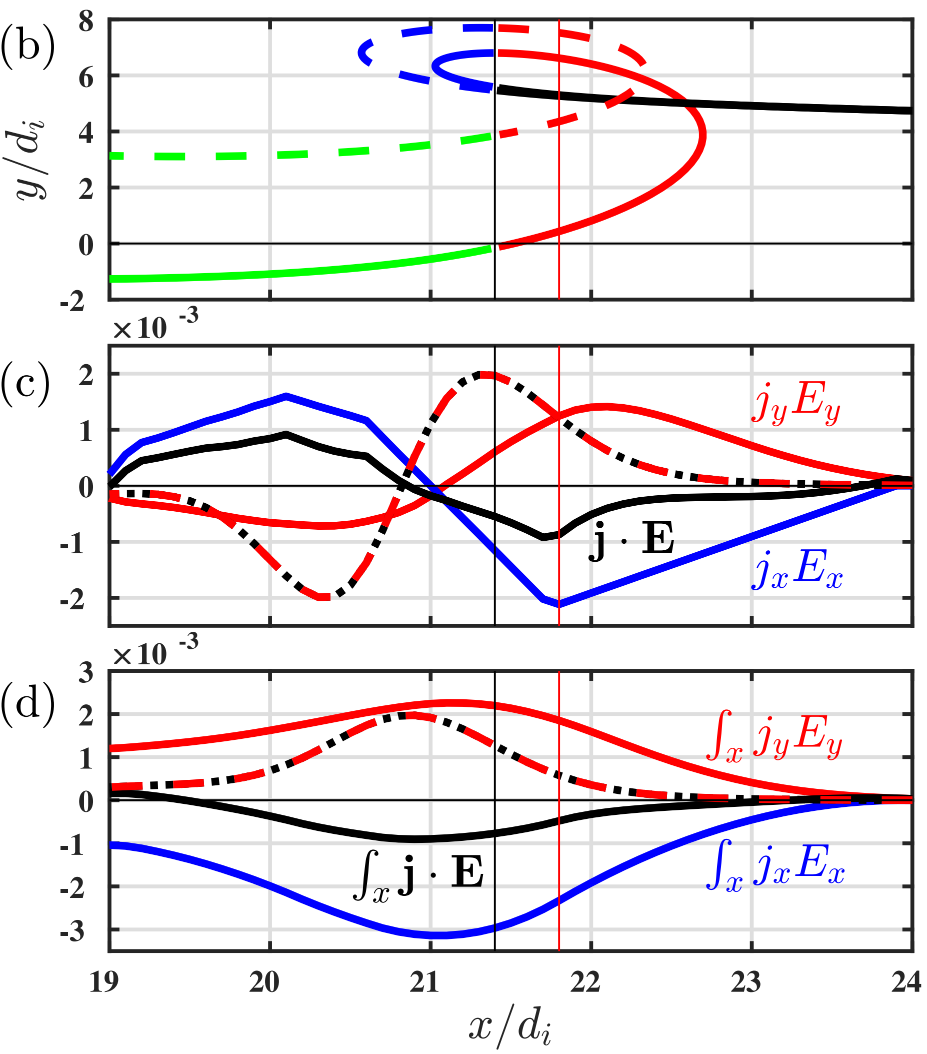


Perpendicular Shock Results for Ions in Gkeyll



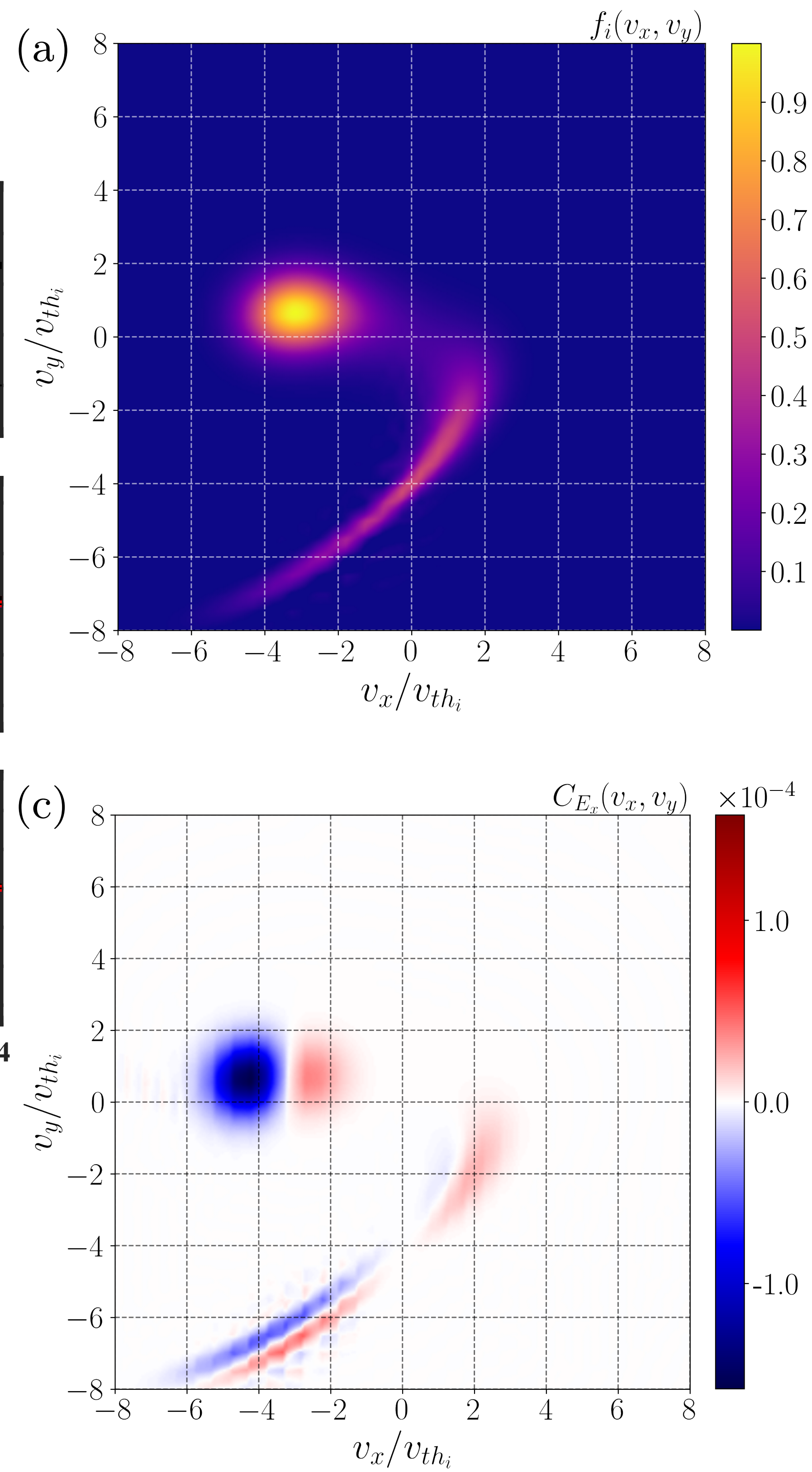
Role of the Cross Shock Electric Field

Ion trajectory and energization

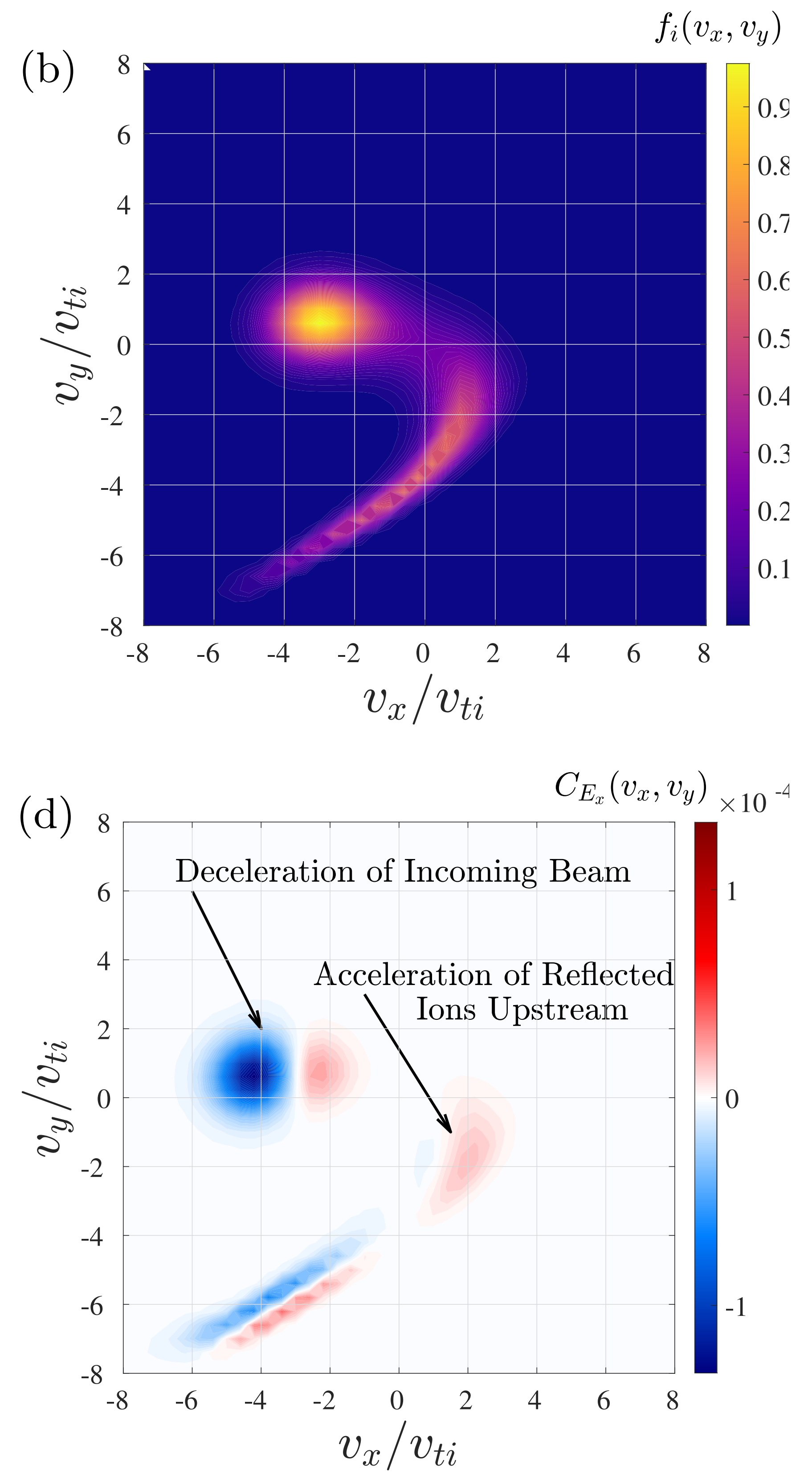


Model with (solid) and without (dashed) the cross shock electric field

Gkeyll

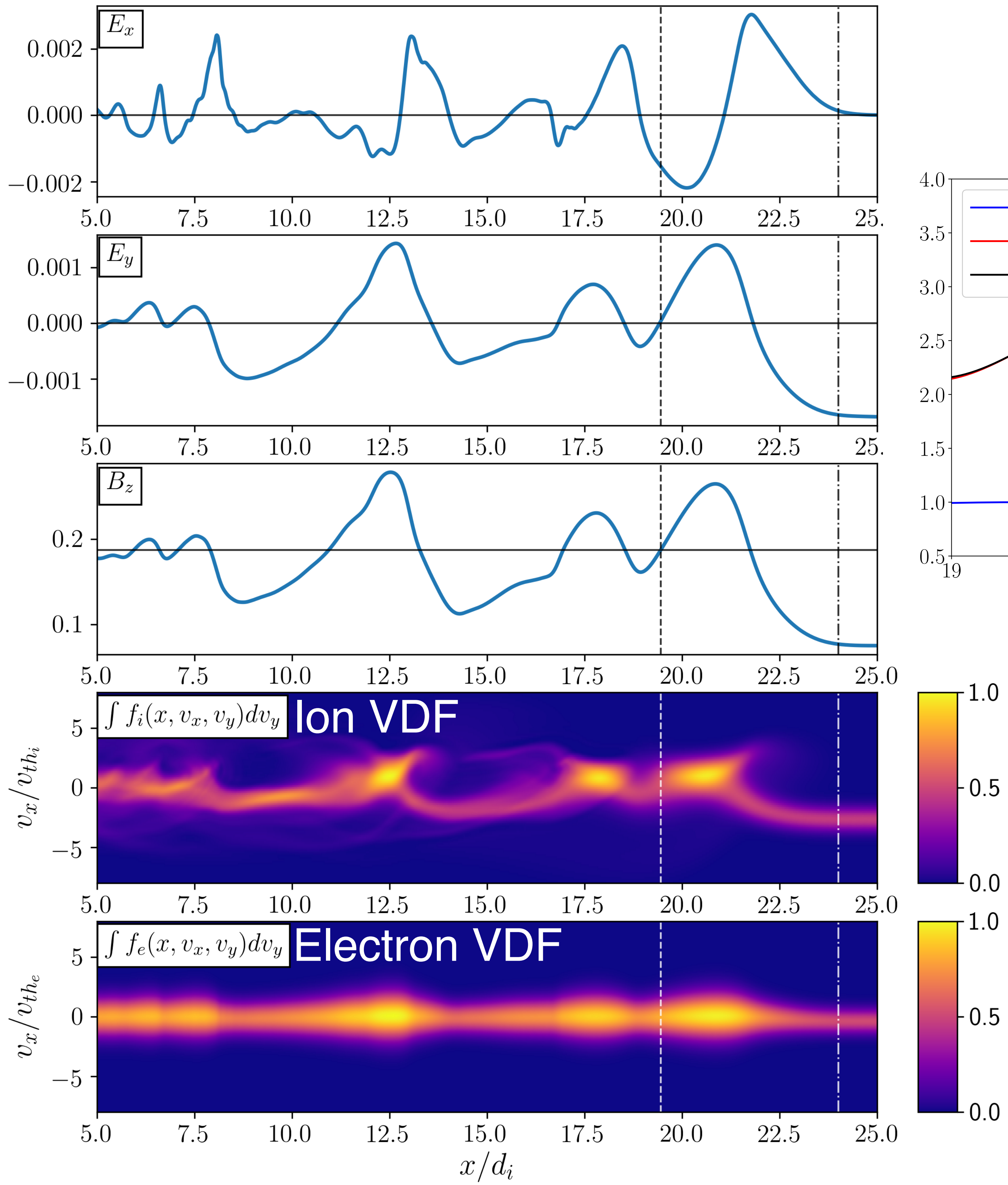


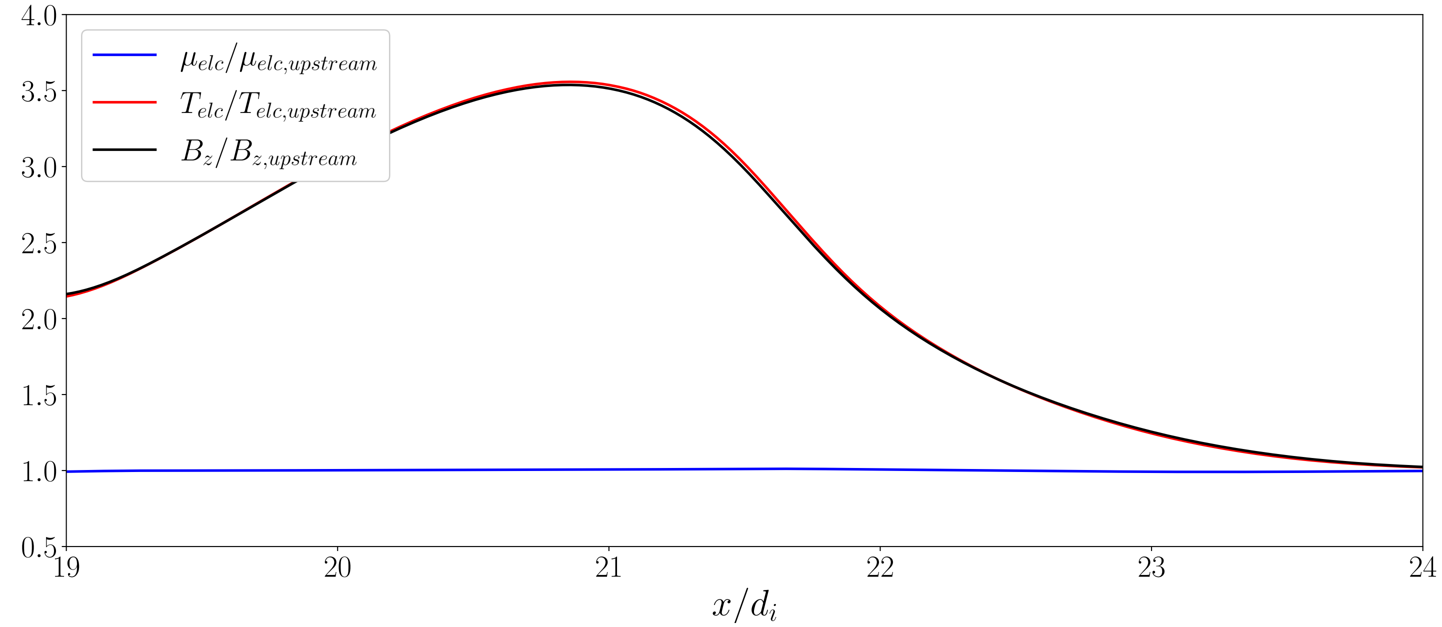
Model with cross shock electric field



Electron energization

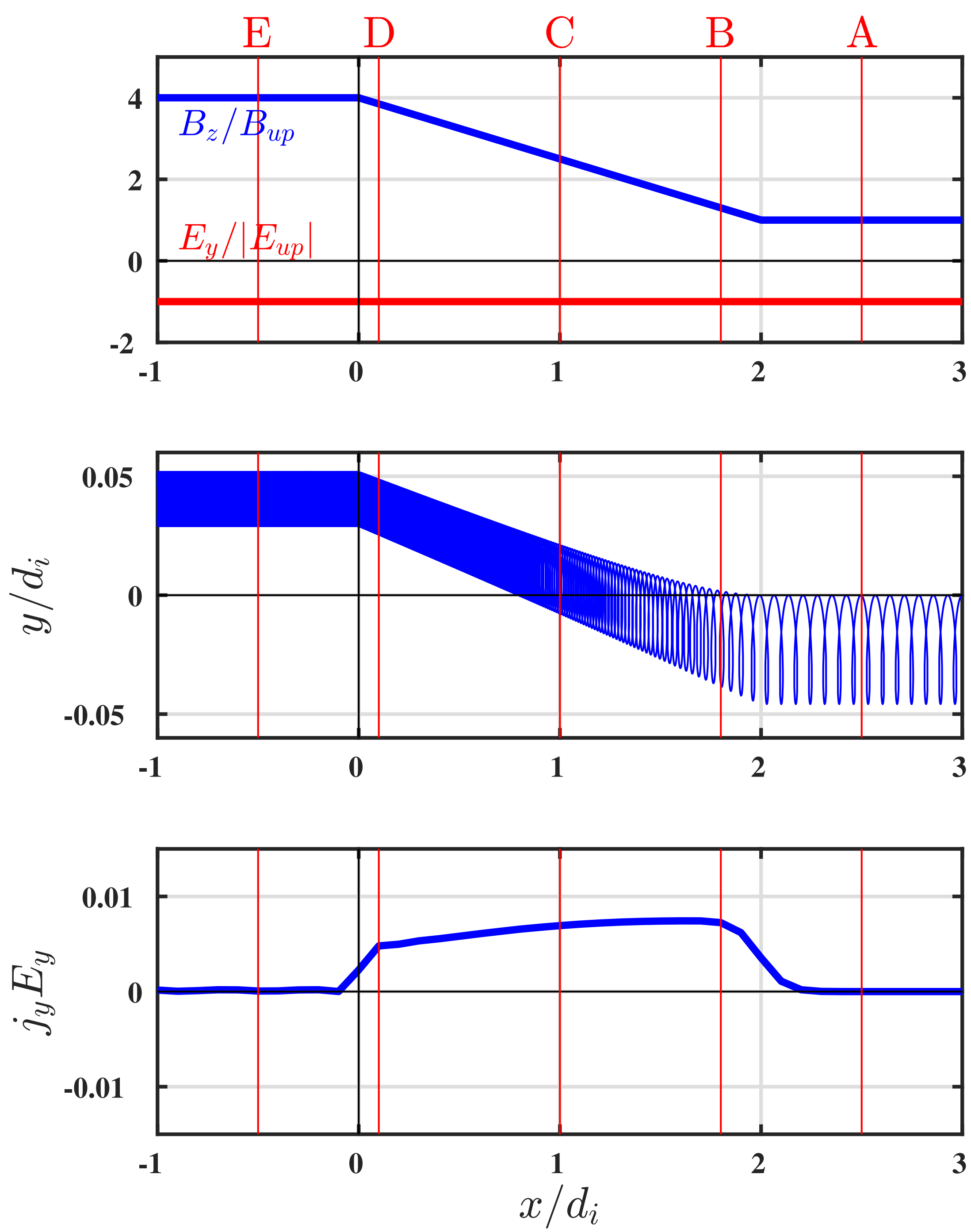
Electron Evolution Through the Shock

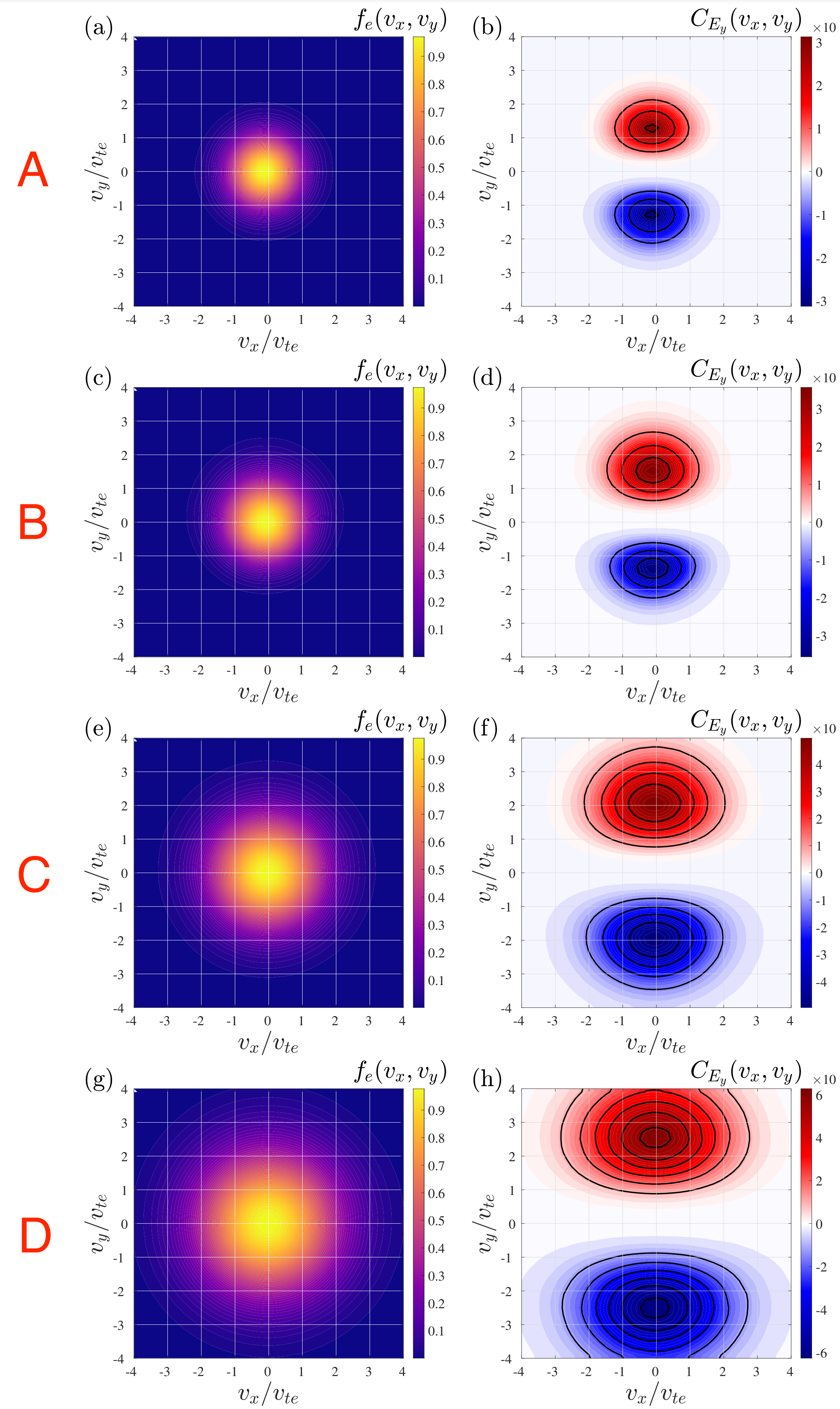




Suggests that the grad B drift is the dominant energization

Simple Electron Energization Model



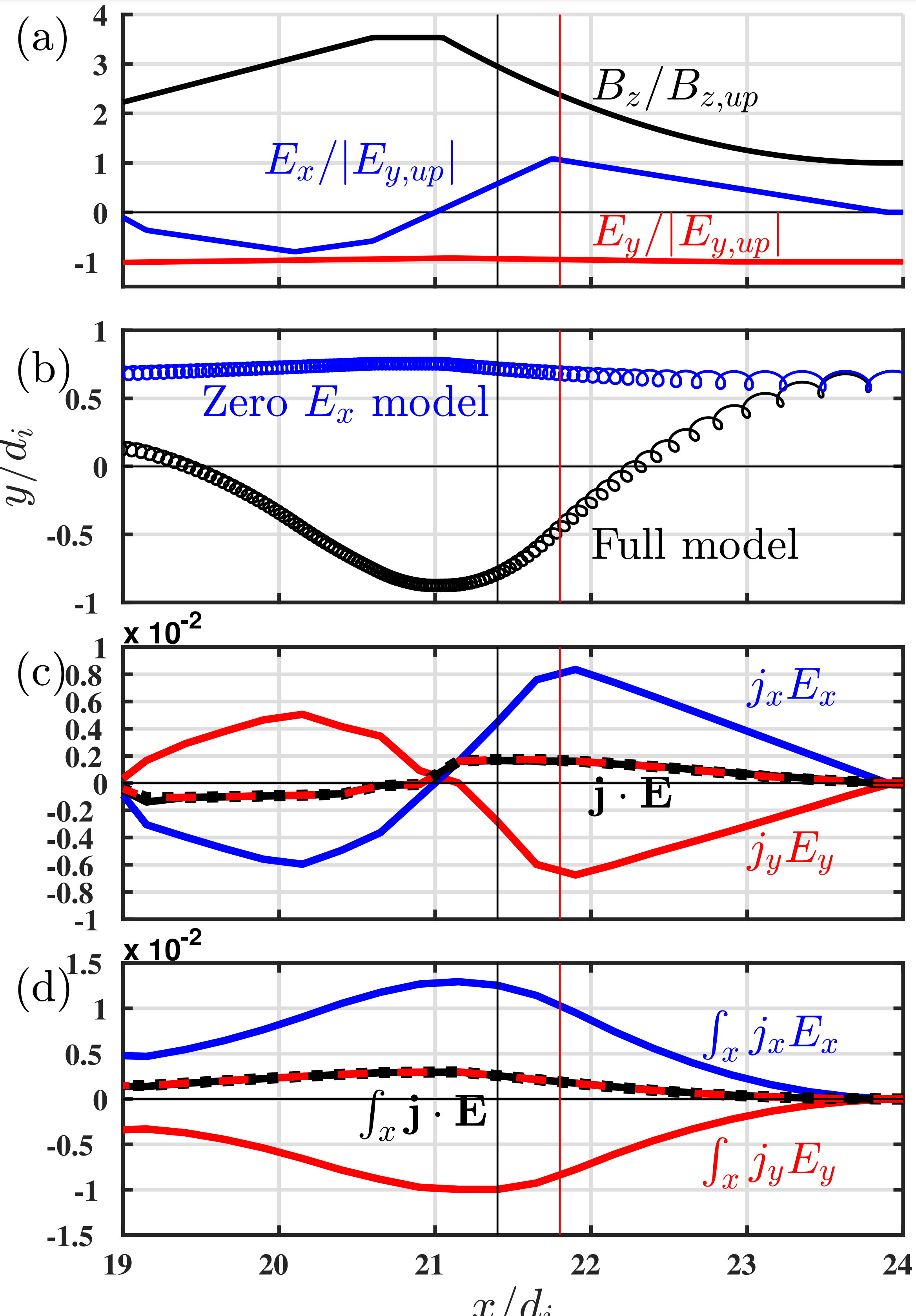


$$\frac{d m_e v_\perp^2/2}{dt} = q_e u_{\nabla B} E_y \qquad u_{\nabla B} = \frac{m_e v_\perp^2}{2 q_e B_z}\left(\frac{1}{B_z}\frac{\partial B_z}{\partial x}\right)$$

$$d/dt = \partial/\partial t + u_x \partial/\partial x = u_{E\times B}\partial/\partial x \qquad u_{E\times B} = E_y/B_z$$

$$\frac{\partial}{\partial x}\frac{m_e v_\perp^2}{2B_z} = \frac{\partial \mu}{\partial x} = 0$$

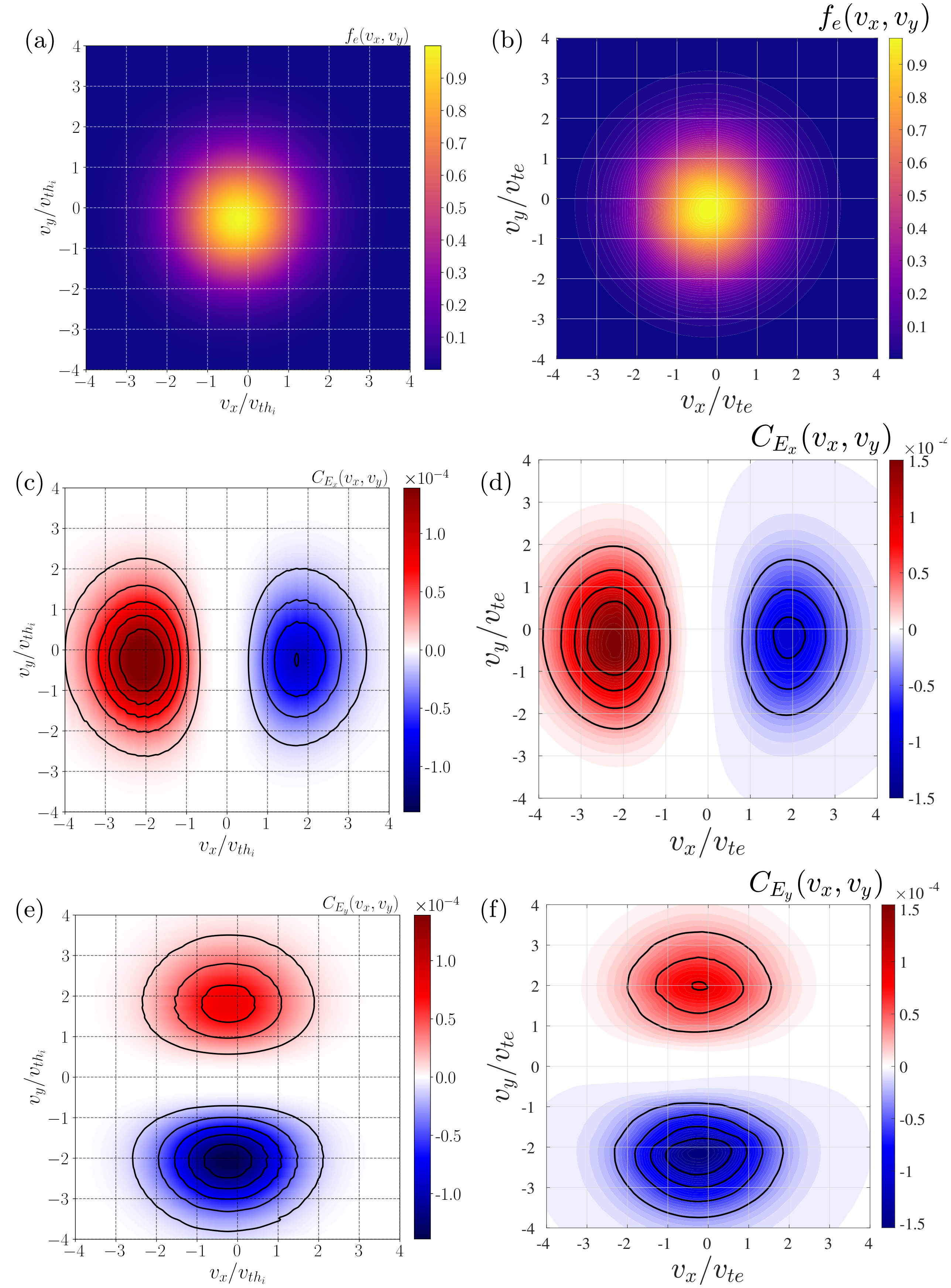
Including the Cross Shock Electric Field



Model with (solid) and without (dashed) the cross shock electric field

Gkeyll

Model with cross shock electric field



Contributions of Drift Components

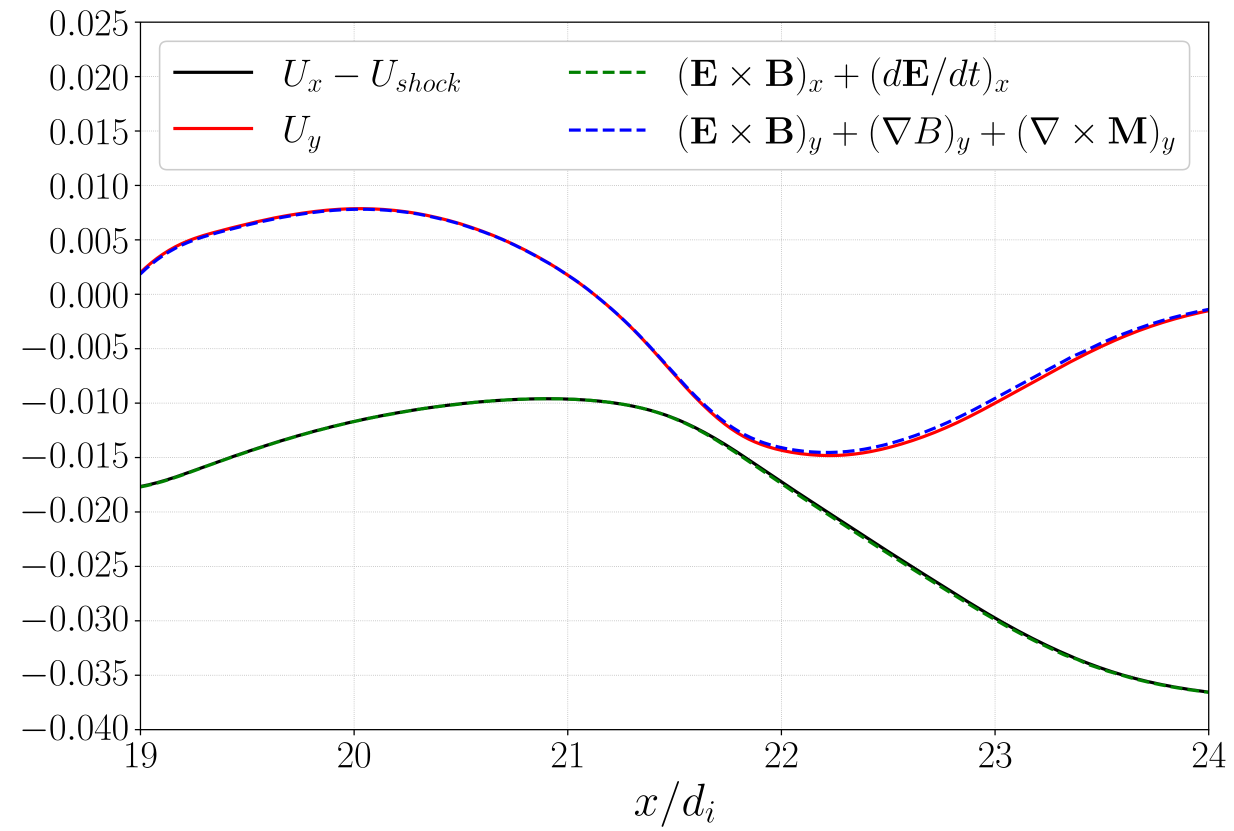
$$u_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

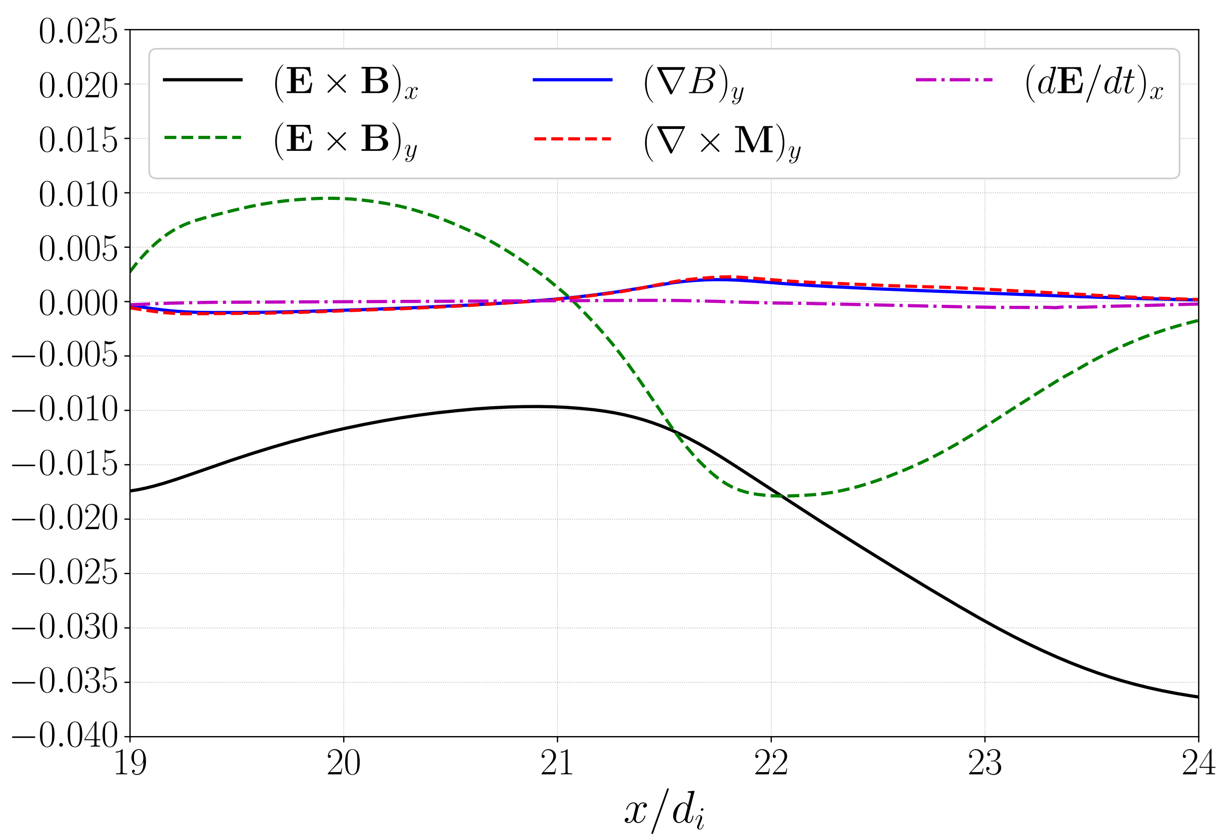
$$u_g = \frac{p_\perp}{qn}\nabla\times\left(\frac{\mathbf{b}}{B}\right)_\perp$$

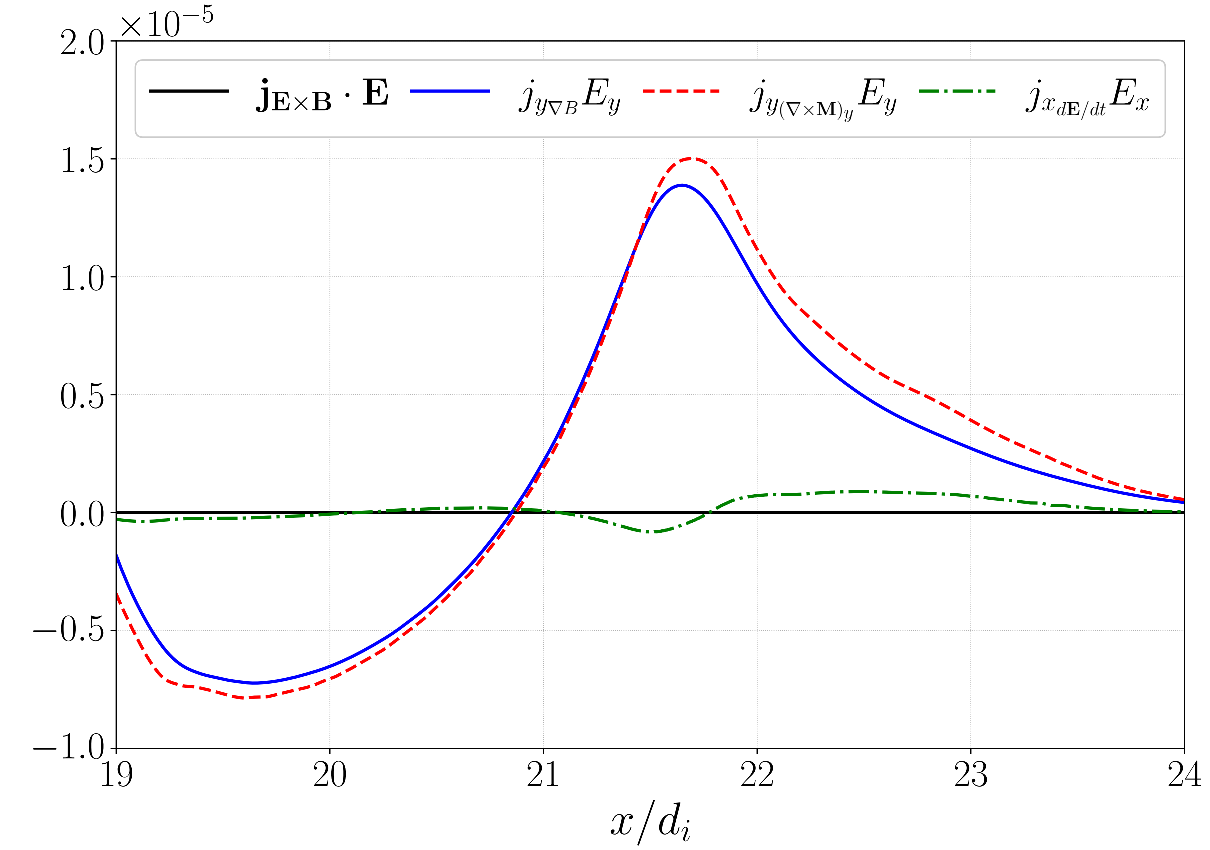
$$u_M = -\frac{1}{qn}\left(\nabla\times\frac{p_\perp\mathbf{B}}{B^2}\right)_\perp$$

$$u_p = \frac{m}{qB^2}\frac{d\mathbf{E}}{dt}$$

$$u_M + u_g = u_D = -\frac{1}{qn}\frac{\nabla p_\perp\times\mathbf{B}}{B^2}$$







Have been considering only the shock rest frame

But, component ExB energization is significant, though it provides no net energization

So, let's transform one component of it away

$$C^*_{E_x}(x, v'_x, v'_y, t) = -q_s \frac{(v'_x - U_{shock})^2}{2} E'_x \frac{\partial f_s}{\partial v'_x},$$

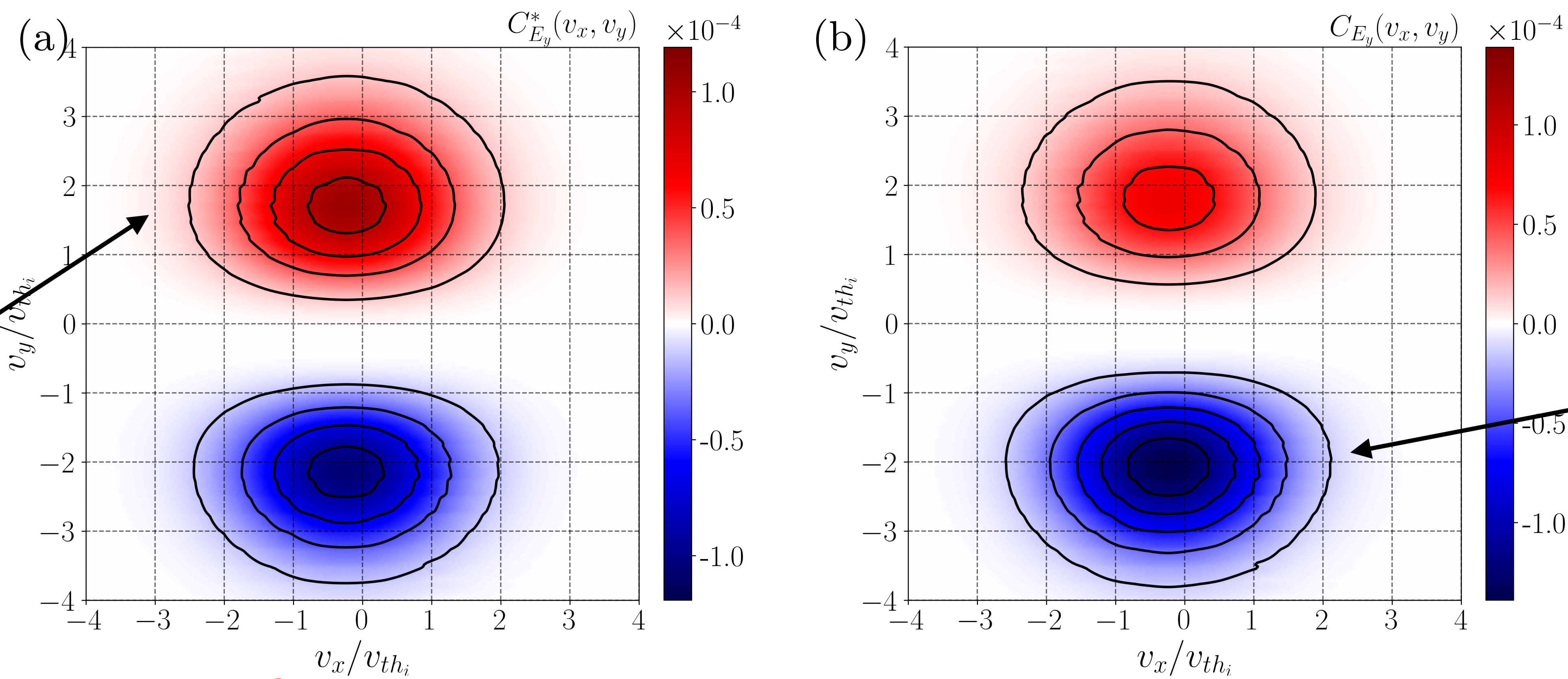
$$C^*_{E_y}(x, v'_x, v'_y, t) = -q_s \frac{(v'_y - E_x/B_z)^2}{2} [E'_y - U_{shock} B_z] \frac{\partial f_s}{\partial v'_y},$$

$$E'_x = E_x - U_y B_z = E_x - E_x = 0,$$

Combined rest frame

Shock rest frame

Net energy gain



Net energy loss

Signature of adiabatic heating

Conclusions and Ongoing Work

- The Gkeyll code provides pristine, high resolution access to full phase space dynamics.
- We have constructed a diagnostic that correlates oscillations in the electromagnetic fields with plasma particles, the field-particle correlation.
- The field-particle correlation analysis technique has been employed to identify the phase space structure of energy transfer via shock drift acceleration and adiabatic heating between the fields and particles in simulations.
- The role of the cross shock potential in energizing particles was found to be minimal.

- Work is ongoing to identify these signatures within in situ spacecraft data.
- Expansion of the simulations to 2x-3v, quasi-perpendicular geometries.
- Additional particle energization processes are also being examined in simulations and spacecraft data, e.g., stochastic shock acceleration, adiabatic heating, and reconnection.